An Evolutionary Approach to Technical Trading and Capital Market Efficiency

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I. INTRODUCTION

There is no greater source of conflict among researchers and practitioners in capital market theory than the validity of technical analysis. The opinions are widely held on each side as well. The vast majority of academic research condemns technical analysis as theoretically bankrupt and of no practical value. This is in sharp contradistinction to the applied world in which <u>every</u> major investment trading house has a technical analysis department. The number of investment newsletters proffering such advice is as astounding as the number of journal articles refuting the theoretical foundations of technical analysis and renouncing its empirical research. There is an almost religious fear with which academic researchers deny the validity of technical analysis.

It is certainly understandable why many researchers would oppose technical analysis on a professional and even personal basis: the validity of technical analysis calls into question decades of careful theoretical modeling claiming that markets are efficient and investors are collectively, if not individually, rational. Within the confines of the Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Lintner, 1965; Mossin, 1966) or the Arbitrage Pricing Theory (APT) (Ross, 1976), it would indeed be heretical to acknowledge the impact of investor behavior which is not perfectly rational. And yet, there is an curious absence in both the CAPM and the APT. Each presumes that investors are mechanisms - capable of arbitrarily complex calculations and blessed (or perhaps cursed) with perfect rationality. Of course, any model is an abstraction from reality, but for models this abstract to be so widely accepted is astounding. There can be no question of it: the CAPM[1] (and to a lesser extent the APT) have dominated every aspect of capital market theory, capital budgeting, and financial economics for decades *without* serious challenge. Over the years, the basic model has been tinkered with and assumptions relaxed and changed. It may be claimed, however, that such minor adjustment misses the point. There has been precious little done subsequently[2] that represents a major departure from a model which, despite its theoretical appeal, has been shown false in a diverse group of empirical tests.

The goal of this paper is twofold, along the lines of the "joint hypothesis." First, the paper addresses the lack of a precise formal theory of technical analysis. Second, the paper addresses a

[1] Here, the "CAPM" referred to is the general body of work on the same theoretical foundations. This includes the original CAPM of Sharpe (1964), Lintner (1965), and Mossin (1966), the Consumption-Driven CAPM of Fama (1970), the Zero-Beta CAPM of Black (1972), the Intertemporal CAPM (I-CAPM) of Merton (1973), and the International CAPMs (ICAPM) of Black (1974), Solnik (1974), and Grauer, Litzenberger, and Stehle (1976).

[2] Treynor and Ferguson (1985) and Shefrin and Statman (1994) are notable exceptions.

test of a theory of technical analysis. Clearly, it would be valuable to have some framework for linking the normative theoretical power of the CAPM to the empirical precision of less rigid asset pricing theories and more realistic adaptive models. This paper approaches the efficiency of markets and the validity of technical analysis from an evolutionary standpoint. Not only do adaptive mechanisms provide an exceptional framework from which to discuss market efficiency and the theory of technical trading, but newly available methods of evolutionary computation (*e.g.* Goldberg, 1989; Leinweber and Arnott, 1995) allow rigorous quantitative testing to be done with such an adaptive theory.

The paper is organized as follows: Section II discusses the issue of capital market efficiency. Grossman and Stiglitz' (1980) and Rode's (1995a) comments on the necessary inefficiency of capital markets are presented, as are Malkiel's (1973) comments on market efficiency and the Efficient Markets Hypothesis (EMH). In Section III, the paper develops a formal theory of technical analysis. Recognizing that past discussion of technical analysis often falls short because the field is so highly fragmented, a testable foundation is laid out which describes why technical analysis is necessary and what it does. In Section IV, the paper describes in detail the evolutionary methodology used to test the theory of technical analysis. Particular emphasis is given to the genetic algorithm model built specifically to address this issue. Section V presents the results of the testing of the model using data on the Standard and Poor's 500 Index from June 15, 1992 to April 28, 1995. Section VI concludes and notes some suggestions for further research. The appendix contains descriptions of the technical trading rules considered in the testing model and the modular genetic algorithm system written in *Mathematica*.

II. CAPITAL MARKET EFFICIENCY

Although Malkiel (1973) would still deny it, the notion that capital markets are perfectly (or even very) efficient is coming under increasing pressure from the weight of a growing body of empirical evidence (*e.g.*, Hawawini and Keim, 1994; *cf*. Appendix A) that would suggest that capital markets around the world aren't nearly as well-behaved as everyone thinks. It is important to define exactly what is meant by "efficiency" before proceeding. As used in this paper, *efficiency* is defined as a market process which causes information to be perfectly reflected in the current prices of securities and new information to be instantaneously integrated. Efficiency essentially means that capital market prices follow a random walk (perhaps with a drift) which is often represented by a geometric Brownian motion. This implies that past securities prices should have no impact on future prices and that statistical forecasting should only work successfully with a frequency that would not significantly distinguish it from random error once transactions costs are included. This is the *martingale* or fair game approach - investors cannot make money by speculating on behavioral tendencies (because all investors are rational) or by arbitraging information flows (because all investors are equally informed).

That these are strong assumptions is undisputed. What is disputed is just how significantly these assumptions are violated in actual market processes. If the assumptions are only weakly violated, then perhaps the process of gradual refinement of the CAPM is appropriate. However, if the violations are as influential and systematic as this paper claims, the only solution to improving capital asset pricing models may be to consider another theory. In fact, Grossman and Stiglitz (1980) prove that it is impossible for markets to be informationally efficient and thus strong-form efficient. They state that if no one performed technical analysis, stock prices would cease to have any meaning at all as reflections of value. In a very interesting paper, Cornell and Roll (1981) solve for the exact size of the proportion of the total market using technical analysis using a solution concept from evolutionary game theory called the *evolutionary stable strategy*.[3]

Research has traditionally considered the market process as a search for equilibrium. This equilibrium is considered to be the optimal point for investors, and thus a Nash equilibrium. If investors did, in fact, optimize, this type of equilibrium may be possible. There is substantial evidence, however, that when decision makers are confronted with new and complex environments, they *cannot* optimize. Newell and Simon (1972) and Simon, *et al.* (1987) have repeatedly shown that bounded rationality governs complex decision processes. Humans are limited by computational processing power (*i.e.*, speed of calculation) and memory (*i.e.*, lack of infinite recall). There is also the issue of domain-specific knowledge. This is especially problematic in the area of financial economics. Cross (1983) remarks that in striving for more and more exacting theories, researchers have drifted so far away from reality that it should be no surprise that contemporary capital asset pricing models have little empirical support.

> The methodological price for this approach [traditional statistical and mathematical decision analysis] has been extremely high, however, for it has become necessary to assume that individuals in these markets can be represented as mathematical statisticians capable of solving specific problems that are often beyond the analytic abilities of professionals in that field. It also requires reliance on the

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[3] For a review of evolutionary game theory and capital market efficiency, see Rode (1995a). Basically, the evolutionary stable strategy, or ESS, augments the traditional Nash equilibrium selection criterion with a population stability requirement. It is this incorporation of the notion of stability which plays a fundamental role in assessing the behavioral impact on the normative solution.

> assumptions that individuals follow optimizing rules of behavior under just those dynamic and risky types of situations for which the assumption of optimization has the least empirical support. (Cross, 1983).

What has been proposed as an alternative to traditional equilibria (Vaga, 1990; Rode, 1995a) is the concept of *stable points*. That is, points which are not optimal, but are local maxima given the constraints on the task environment faced by the decision makers. Thus, capital markets reach an "equilibrium" of sorts, but not anything near what a traditional equilibrium would be considered. The outcome is not optimal. Traders are not doing their absolute best, but rather the best they can given the circumstances at the moment of decision. In normative equilibrium models, such suboptimal behavior would be driven out. In actuality, however, *everyone* is similarly constrained and thus there is no one to drive out the behavior. Further, equity prices cannot be arbitraged in a manner similar to derivative prices or fixed income securities. There is no one certain value for an equity and therefore, prices are determined, within obvious value constraints, primarily by supply and demand. It must be made clear, however, that the notion that securities prices are determined by supply and demand is rather controversial, despite it's market-oriented nature.[4]

Hawawini and Keim (1994) present an outstanding synthesis of tests of market efficiency. This paper is not so much interested in that markets are not efficient, but rather on *why* they are not efficient. Markets are not efficient because investors are prevented from optimizing. Due to these constraints on optimization, investors resort to using heuristic rules to guide their decision making. It is easy to see that investors who are unconstrained by cognitive limits on memory capacity and are not pressured by time could make optimal decisions. Humans, however, are substantially limited in those capacities. The predominant decision environment for humans is one in which decisions must be made with incomplete and even unknown information and under time pressure. In these environments, humans use heuristics which they adapt over time to make reasonably good decisions in complex problem spaces. Rule-based thought and inductive logics are incomplete, and thus are confined to suboptimality, yet, it many situations, the complex evolution of decision processes according to simplifying heuristics produces significantly good results (Rode, 1995b).

Markets are, therefore, necessarily inefficient. This has an important conclusion: the

[4] As efficient market theorists would have it, if the value of a security deviated too greatly from the value of the underlying firm, an investor could make an arbitrage profit by, if the stock was undervalued for example, purchasing all the stock and selling the assets of the firm simultaneously. It should be obvious that this is impossible except for enormous deviations from true value, and only within capital availability constraints in the markets.

CAPM and APT are not incorrect, they are merely conditionally correct. *If* investors behave rationally, then the CAPM and APT are appropriate models for determining market equilibrium. However, if investors are quasirational, then the CAPM and APT cease to be correct determinants of market equilibrium because they ignore the systematic errors that quasirational behavior causes. This view is very similar to that proposed by Vaga (1994) concerning the *coherency* of markets. Vaga felt that markets transition between states governed by rationality (and the thus the CAPM) and states governed by irrationality (and thus were not truly random).[5] It was the transition period between these states, called a *coherent market*, a period governed by quasirationality, that Vaga felt predictive powers were greatest. Although Vaga's theory claimed that the quasirationality of the transition state was characterized by chaotic tendencies, no assumption of chaos will be made here - directly. It is clear, however, that the issue of *complexity* is vitally important to such a formulation. Complexity and chaos are not synonymous and this paper will solely address complexity (see Gell-Mann (1994) and Arrow (1994) for further discussion on the characteristics of complex systems).

The conclusion that may be drawn from this research is that markets are not efficient and that they are not efficient because investors *cannot* behave optimally. The task environment facing investors is one in which there are substantial constraints on the information processing time allowed (for floor traders, decision must be made immediately after new information is released). There is also a continual abundance of new information made available. This flow of information easily exceeds investors' abilities to process it completely. Surely Simon would consider this the *ne plus ultra* of constrained task environments: immediate, vastly complex, and critically important decisions must be made repeatedly. The market deviates from efficiency in ways that traditional equilibrium models cannot begin to incorporate.

III. A THEORY OF TECHNICAL ANALYSIS

This section outlines a theory of technical analysis as an alternative to the perfect equilibrium theories discussed previously. Because investors are prevented from making optimal decisions they must use heuristic rules to guide their decision making. This does not mean that their decision making is random or that it is doomed to failure. In many cases, humans using heuristic

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[5] The reality of these states not being "perfectly random" is measured by the fact that they have a fractal dimension of between 1.5 and 2. Specifically, both Fama (1965) and Peters (1994) have found that the fractal dimension of the S&P 500 Index is approximately 1.65. A fractal dimension of between 1.5 and 2 implies a persistent or self-reinforcing series.

rules can perform quite well (*e.g.,* Albers and Laing, 1991; Dworman, Kimbrough, and Laing, 1994). This is the foundation of the theory: technical trading rules represent efficient heuristic rules which can be used to make reasonably good investing decisions. The object of technical analysis is to predict a complex time series with one which is easier to calculate and forecast. This is exactly the essence of simplifying heuristic behavior: substitution of the less complex for the intractable.

Thus technical trading represents a "rational" choice for boundedly rational investors. Technical trading can allow investors to make reasonably well-informed decisions with relatively small information processing costs. Various work has been done on the predictive power of technical analysis (*e.g.,* Neftci, 1991; Blume, Easeley, and O'Hara, 1994) and the results have generally been supportive of the technical rule approach. It is clearly conceded that technical analysis doesn't *not* produce optimal results. Neftci (1991) stresses that Wiener-Kolmogorov prediction theory and vector autoregressions should do a better job of forecasting such time series. However, such analysis is extremely complicated and can be rather computationally intensive. Also, more importantly, there is an implicit assumption that the market process is linear. Wiener-Kolmogorov theory ignores information contained in the higher-order moments and linear models in general are ill-equipped to deal with nonlinear time series (of which there is mounting evidence for the securities markets).[6] For a floor trader who must make a decision instantaneously, use of the such techniques is simply not feasible.

Sears and Trennepohl (1993) identify five factors as the principal reasons why technical analysis would work:

1. The price of a security is determined solely by its supply and demand.
2. Prices tend to move in trends that persist for an appreciable time.
3. Changes in trends are caused by changes in supply and demand.
4. The patterns of trends tend to repeat themselves over time.
5. Supply and demand is governed by both rational and irrational factors.

What is most obvious in these factors is the presence of persistent, similar behavior. In earlier attempts to compensate for the evidence of market efficiency, it was proposed that such irregularities would disappear in the aggregate as one irrational act canceled out another. It appears

[6] Interestingly, Neftci also notes another reason for the popularity of technical analysis. That is, if prices indeed were martingales, vector autoregressions would produce "trivial" looking forecasts, such as $X_{t+\tau}=X_t$ where $\tau=1,2,\ldots$. Forecasters, Neftci then claims, face significant pressure to avoid making such "unimpressive" forecasts and "irrationally" choose to use technical analysis in order to generate "nontrivial" forecast. Of course, this paper attempts to show that the use of technical analysis is not necessarily irrational, but this insight remains enlightening.

today, however, that the interaction among various quasirational agents does not eliminate inefficiency. In fact, it may serve to *sustain* inefficiencies by promoting further complexity.

Technical analysis represents a prudent method for coping with increasingly complex investment environments. However, simply stating that technical trading rules are a method does not solve the problem. In fact, it creates two new ones: *which* trading rules and *how* are they applied? One of the problems with previous tests of technical analysis is that the forecasting ability only seemed to persist for short time periods with any one technical model. It was assumed that because one technical model did not hold permanently, technical analysis had no value. It should be clear, based on what has been stated above, that *that* kind of technical analysis indeed has no value. Merely supplanting suboptimal static rules (single indicator, static technical analysis) for optimal static rules (CAPM-based strategies) can only guarantee poor performance. The key to any successful strategy is to adapt and change over time and to *learn* to accommodate new environments. Tests of technical analysis can fail because: (i) technical analysis doesn't work (the markets are indeed random), (ii) the wrong technical rules were considered, (iii) the rules were misapplied.

Any technical trading model must not only know the correct trading rules for a particular time period, it must also know when to change those rules and what to change them to. Further, it is clear that no one technical indicator is omniscient. Investors look at more than one indicator and often many indicators on a regular basis to guide their investments. Section IV discusses the specific representation of the decision environment in the context of a test of this theory, but a brief outline will be presented next. Consider the case of an investor approaching an investment decision.

The time constraint is such that formal analysis is implausible. The investor knows that among the set of technical trading rules at his disposal, there is some combination which currently has predictive value. However, he does not know what combination that is or how to update such a mechanism to reflect the inevitable changes in the importance of the various rules. In technical terms, he cannot observe the behavioral dynamics of the market in any meaningful manner. A technical trading system must be able to select the rules and guide their use throughout the future of the model. Such a process has been termed the Consensus[7] mechanism. Investors consider a set of various technical trading rules which remain constant. What the mechanism creates are metarules which determine which technical rules are used in which environments. Different sets of rules may

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[7] see Rode, D. (1995). Consensus: A Heuristic Algorithm for Equity Portfolio Management. Unpublished manuscript.

have greater predictive power in rising markets than in declining markets. Various combinations of rules may interact to increase the predictive power of the system. Each of these events is likely, but unobservable by the investor. The investor's problem is that there appears to be no easy manner by which to select the appropriate metarule. The "consensus" is involved in the final decision. The investor selects the decision (buy the security or hold cash) which represents the informed consensus of the other rules in a procedure similar to weighted majority rule voting.

For example, consider the case of an investor with three rules, $\{n=3 \mid x,y,z\}$. Conditioned in some manner on their performance (this will be discussed in detail in Section IV), each rule has a "weight" $\{\omega_x, \omega_y, \omega_z\}$. The output of each rule is then mapped onto the binary relation $\mathscr{F}_i = [0,1]$ in accordance with the properties of the technical rule.[8] If $\Re$ represents the result of the technical rule system, $\Re$ is calculated such that

$$\mathscr{F}_{i \in \{x,y,z\}} \rightarrow [0,1]$$

$$\Re = \frac{\sum_{i=1}^{n} \omega_i \mathscr{F}_i}{\sum_{i=1}^{n} \omega_i}$$

$$\text{Return} = \begin{cases} r_m & \Re > 0.50 \\ r_f & \Re \leq 0.50 \end{cases}$$

where r_m represents the return on the security and r_f represents the riskless return earned by holding a money market instrument (here represented by short-term U.S. Treasury bills).

Section IV discusses the method by which these rules are weighted. The insight this paper offers is a method for calculating the weights of the various rules dynamically using evolutionary computation: specifically, a genetic algorithm (Goldberg, 1989). This model is based on the premise that the majority of rules which have predictive power are known. This may be a faulty assumption and it should be noted that this approach is explicitly different from the work

[8] Technical trading rules are calculated on a variety of indices and scales which makes the normalization accomplished by the mapping necessary. In general, however, each technical trading rule's output can be classified as bullish (buy security, represented by 1) or bearish (hold cash, represented by 0).

done by Allen and Karjalainen (1993). Allen and Karjalainen use genetic algorithms to search for new technical trading rules. Although they are successful in finding rules which do indeed have predictive power, this paper would claim that such research does little to advance the cause of technical trading and does much in the way of aiding its detractors. Their approach may crudely be construed as nothing but intelligent data fitting. The literature is rife with systems which will exactly fit any arbitrary number of data points (see Leinweber and Arnott, 1995). The fact that their system had predictive power is impressive, but does not substantiate or have the ability to substantiate the *theory* of technical analysis. Most of the widely used technical trading rules that currently exist have at least some theoretical premise behind them. Thus, the model used in this paper only uses rules which currently exist and have provable foundations. This model does not create technical rules - it creates metarules. That is, it specifies *how* these existing technical rules are to be used. This methodology more precisely accommodates how technical analysis is used in actual trading and investing.

This section has outlined specifically the theory of technical analysis: technical rules are heuristic rules for investing and investors[9] use heuristic rules because they are substantially constrained from making optimal investment decisions. These rules are updated according to the Consensus process which, through the use of a genetic algorithm, is able to discern the previously unobservable process of behavioral market dynamics by which the value of the existing rule portfolio changes (the metarules). Additionally, this section has outlined a model by which to test the theory and how such a model differs from previous attempts by incorporating a dynamic adjustment process to existing rules and not by generating somewhat arbitrary new rules.

IV. THE EVOLUTIONARY DYNAMICS OF TECHNICAL TRADING

Upon consideration of the proposed problem, genetic algorithms seemed particularly germane to the specifications of a technical trading system. These methods, because of their ability to explore a vast search space (here, estimated as $128^7 \approx 10^{14.75}$) in an efficient and robust manner, provide an excellent way to contemplate a market system which cannot be accurately modeled. The number of possible combinations of rules and the importance assigned to each yields a problem space which is not tractable. Furthermore, because there is no finite or definitive answer as to the proper use of

[9] It is important to note that, much like the Grossman and Stiglitz (1980) paper, not all investors will use technical analysis. Clearly, those investors who are not constrained by time or other limitations will prefer to do further analysis before investing.

technical trading rules, genetic algorithms can be constructed to evolve a plausible or best fit solution for their use. In addition, the ability of such a program to find a weighting or combination of rules which outperform a simple buy and hold market strategy provides vital evidence to attest to the validity of using these rules in market prediction.

In order to substantiate the value of technical analysis, we have placed real–world constraints on our work. In this fashion, this research takes a step away from previous work (Allen and Karjalainen, 1993) done using genetic techniques which use only off–line data and therefore merely prove that above market returns *could* have been achieved using technical rules over a series of historical data. This previous work does little to advance the validity of technical analysis as applied to day–to–day investment strategies. The first constraint on a real–world application is its running in realtime. In this case, realtime means that the data needs to be analyzed and reach an applicable solution between the close of the market on one day and its opening on the next. This is necessary so the model can actually produce prescriptive forecasts for day–to–day trading activities before its results are mere historical observations. It seems appropriate that the system need prepare its response one hour before the market's opening. This hour will allow the traders time to make adjustments to their positions not accounted for within the model. Several examples of these conditions might include liquidity constraints, macroeconomic events which could plausibly distort the expected progression of the market, or the receipt of crucial, price–affecting information on a firm or industry's performance. For this application, which uses the Standard & Poor's 500 Index of common stocks, it has been estimated that the final revision of market data is made accessible at approximately 6:00 PM (Eastern Standard Time) while the market opens at 9:00 AM (Eastern Standard Time). This gives the system fourteen hours to completely reprocess the new set of relevant data. Naturally, on weekends and holidays more time is available for the model to run. However, for simplicity the strictest constraints have been assumed.

An individual string models the heuristic process by which technical trading is most commonly conducted. It is composed of both a primary or prognosticatory indicator and a group of seven other rules which in composite serve to confirm or deny the prognosticatory rule's recommendation. In order to insure that the rules applied would maintain a valid theoretical foundation, a set of eight commonly used and generally accepted rules are fixed within each individual (see Appendix A). Variations between individuals occur in both which rule is designated as prognosticatory and by the relative weighting of the other seven rules within the confirmation process. The highest possible weighting for any one confirmatory rule is 128 which can be represented in seven bits in binary coding. Therefore, each individual is composed of eight groups (one prognosticatory rule plus seven confirmatory rules) of seven bits each. To prevent double weighting, the prognosticatory rule is assigned a zero for each of its seven bits. The system is constructed to permit only one prognosticatory rule per individual.

In the evaluation of an individual, it is important both that the confirmatory rules agree with the prediction of the prognosticatory rule and that the prognosticatory rule's output equals the actual movement of the market. Furthermore, it must be considered that a rule that is correct when it has a higher weight and thus more influence deserves more consideration than the performance of a rule given a lower weight and therefore less influence.

With these factors in mind, a string and its rules are evaluated for fitness as follows: A confirmatory rule is examined. If the output of this rule (0 indicates to hold cash, 1 indicates to buy the security) is equal to both that of the market and that of the prognosticatory rule, it is then given a value of one. If these numbers are not in agreement, then it is assigned a negative one. Finally, this one or negative one is multiplied by the genetically evolved weight of the rule within the confirmatory process. As previously noted, weights can vary from 0 to 128 as represented by the 7–bit binary representation of each rule within the string. Each weight is thus indicative of the confirmatory rule's strength in the voting process. The evaluation procedure examines one confirmatory rule at a time and then sums the given values of each rule. Therefore, a string can have a maximum fitness of 896 (the summation of the values of seven rules all given a 1×128 weighting). In addition, it logically follows that the minimum possible fitness of an individual is equal to –896. Finally, to avoid the use of negative numbers, 896 is added to all computed fitnesses to give a possible fitness scale from 0 to 1,792. Maximum system fitness is $16 \times 1792 = 28{,}672$.

In generation zero, after all fitnesses have been calculated, the system selects two strings for each of the eight prognosticatory rules. These two strings represent the highest fitness string indicating "buy" and the highest fitness string indicating "hold cash" for each prognosticatory rule. The comparable selection conducted in subsequent generations, however, does not require an equal representation of rules but merely selects the sixteen rules having the highest fitnesses over all generations. This grouping of rules then enters the confirmatory process which can be most easily understood as a three stage mechanism: the elimination of inactivated states, elimination of unconfirmed rules, and finally the comparison of relative fitness and the subsequent selection of a trading metarule. During the first eliminations,the current information is added to the system in order to compute what the actual best market response (buy or hold cash) for the previous day's performance was. For example, when a population is composed of an equally distributed representation of the entire set of market possibilities (as in generation zero), eight of the sixteen possible states can immediately be voided since the prognosticatory rule will now have a defined output. This will reduce the matrix to one possible response for each prognosticatory rule. In the next part of the confirmatory process, a "check" is done to make sure that the majority recommendation of a string corresponds to the counsel of the prognosticatory rule. This test is conducted by first multiplying the weight of each rule in the string by its recommendation to buy or hold cash (designated by 1 or 0). The results for each confirmatory rule are then summed and divided by the sum of the weights for the string. A value between zero and one–half instructs one to sell while a number between one–half and one indicates the recommendation to hold. If this directive does not match the output of the prognosticatory rule within the string, then the entire rule is eliminated. Finally, of the remaining rules, the fitnesses of each are compared and the rule string with the highest fitness is selected as the weighted majority winner. This rule represents the informed forecast of the system.

As in other genetic algorithm systems (*e.g.*, Goldberg, 1989), the vital genetic component operates through a combination of reproduction, mutation, and crossover. While reproduction insures the preservation and replication of useful genetic material in proportion to the fitness of an individual, crossover allows for new combinations of this material to be created in order to produce new possible solutions for the problem at hand. Finally, mutation insures the introduction of new and potentially useful genetic material. Upon initial contemplation of the proper design of this system, preliminary rates of crossover and mutation equal to 0.5 and 0.001 were considered in adherence to suggestions made for genetic algorithms employing technical trading techniques (Bauer, 1994). These rates were then tested and confirmed as possibly optimal and certainly effective by multiple runs of the genetic algorithm using stochastic data.

The initial stage of creating a new generation is reproduction. The group of individuals possible for selection for reproduction are the sixteen above as indicated within the confirmation process. Using a roulette wheel, strings are reproduced in proportion to their relative fitnesses. Each string is awarded a "space" on the wheel with a size equal to its fitness divided by the sum of all of the fitnesses of that generation's population. This method follows the seemingly standard evolutionary construct which affords individuals with higher fitnesses a greater chance to perpetuate more of their genetic material through further generations.

Initially, a pair of strings is selected from those chosen for reproduction. Then, by the flip of a probabilistic coin, these strings are selected with probability of p_{cross} to undergo crossover. If designated for crossover, the exact point is then chosen by the roll of an eight-faced die. Finally, each weight is “cut” asunder at the crossover point and exchanged with its mate. For example, consider two strings a and b[10]:

$$a = (0110101)(1011010)(1011010)(1011010)(0000000)(0101010)(0010010)(0111011)$$
$$b = (0001100)(1100101)(1000000)(1110101)(1110110)(0000000)(1110111)(0010010)$$

These individuals have been selected for crossover at bit two. Therefore, crossover will occur two bits over within the weightings designated for each rule. The result will be as follows:

$$a = (01|01100)(10|00101)(10|00000)(10|10101)(00|10110)(01|00000)(00|10111)(01|10010)$$
$$b = (00|10101)(11|11010)(10|11010)(11|11010)(11|00000)(00|01010)(11|10010)(01|11011)$$

Then, the bits in the position of the initial prognosticatory rule of each string are returned to zero.

$$a = (01|01100)(10|00101)(10|00000)(10|11010)(00|00000)(01|01010)(00|10010)(01|10010)$$
$$b = (00|10101)(11|11010)(10|11010)(11|10101)(11|00000)(00|00000)(11|10111)(00|10010)$$

This final step preserves the proportion of the population with a given prognosticatory rule as selected during reproduction. Furthermore, it provides a relatively simple insurance that string structure will remain consistent throughout the genetic process.

Mutation is also conducted using a probabilistic coin flip. Each bit within a weight string is examined and then chosen with probability of p_{mutate} for mutation. If selected, a bit is flipped: a bit previously equaling one is changed to zero and vice versa. Although this is perhaps not the most computationally efficient method for this procedure, this issue is not necessarily relevant since the process is effective and computing time in this operator is proportionally small in comparison with the calculation of the fitness function. Following mutation, mutated weights of the prognosticatory rule are returned to zero.

Finally, the newest generation "is born" and is ready for evaluation and confirmation. Following the input of a new day's data, the system is able to readapt to the changing conditions of the market and to find the best weighting of the technical indicators as a response to this new information. In

[10] Parentheses are indicated to show the division between the bits representing each rule. The rules are always considered in the order of one through eight with the numbers within the first parentheses referring to the weight of rule one, those in the second referring to rule two, etc. The prognosticatory rule is designated by all zeros for its weighting bits. In this example, string a and string b have rules five and six respectively as their designated prognosticatory indicators.

this manner, the model is able to provide a trader with a new market strategy which is updated on a daily basis. The computational efficiency offered by a genetic algorithm affords the daily fine-tuning of the magnitude of an individual trading rule's impact on the trader's final decision. In this fashion, the system should allow the trader to capitalize on any opportunities which potentially could arise from the application of technical trading rules.

V. TESTING THE GENETIC TRADING SYSTEM

It must first be noted that the tests as presented here are substantially insufficient. Because the model used here, referred to as the Genetic Trading System (GTS), was built explicitly for realtime use, any tests would have to incorporate these time requirements. Consequently, the true benefits of this genetic approach must be measured in hours of computing time. Because of the time constraint for completing this research, testing was not able to be as comprehensive as would be necessary for a statistically significant conclusion. Adequate tests would have to be performed over a long time period, perhaps a year. The system is designed to make maximum use of the available computing time (14 hours overnight). Thus, simulating actual processes over n trading days would require at least $14n$ hours. The tests presented here represent two types of tests: short-term large generation testing, and longer term, small generation testing.

The initial testing of consisted of simulating five days of actual trading by running the GTS for 500 generations per day.[11] The crossover rate was set to 0.50 and the mutation rate was set to 0.001. This resulted in a cumulative five day return of 0.0889%. By comparison, the market return was -0.4436% over the same five day period. The standard deviation of the GTS return was 0.0100% and the standard deviation of the market return was 0.3630%. The GTS also showed an increase in system fitness ranging from approximately 10,000 to 16,000 on average. Figures 1a through 1e illustrate the evolution of the rules. Figure 1f presents a plot of the returns. The heavy line represents the return of the GTS and the thin line the unmanaged S&P 500. According to the Henriksson and Merton (1981) market timing regression technique, the GTS had market timing ability, although it was not statistically significant ($p = 0.45$). The system was correct 3 out of five days, for an accuracy rating of 60%. Still, because of the extremely small sample size, the results cannot be statistically significant.

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[11] For comparison, this required approximately 10 hours on an 80Mhz Power Macintosh or 90Mhz Pentium PC.

Figure 1a

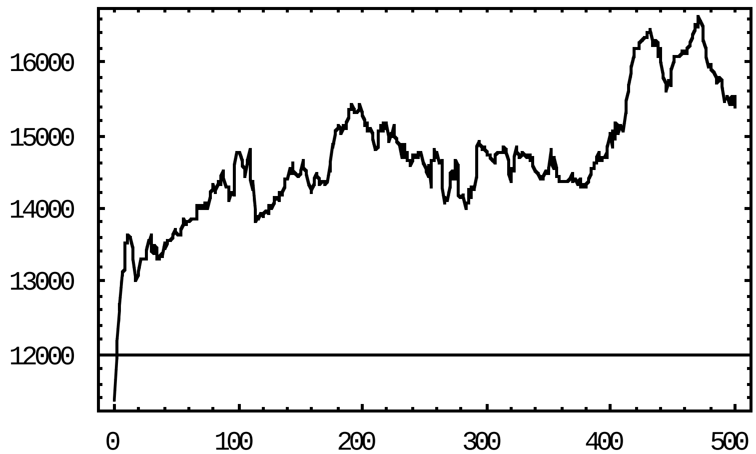


Figure 1b

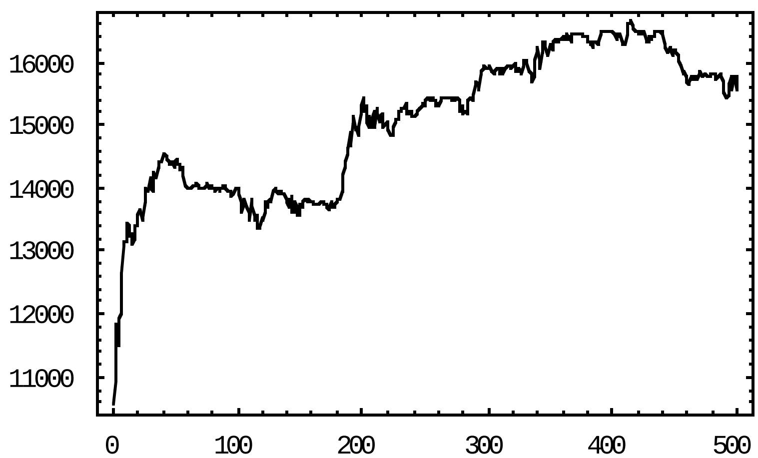


Figure 1c

Figure 1d

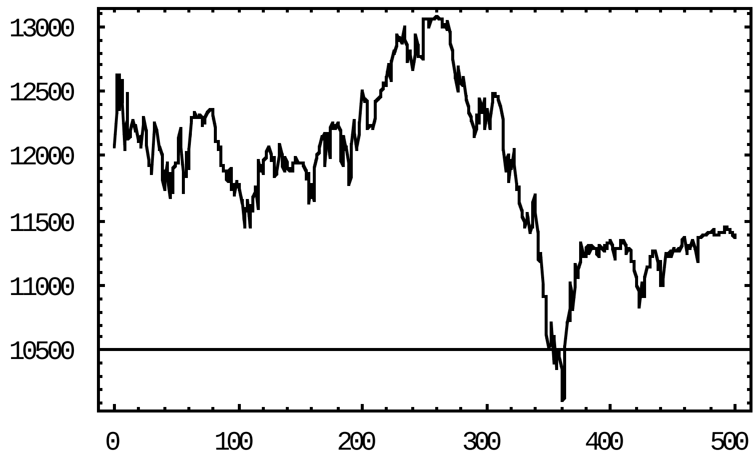


Figure 1e

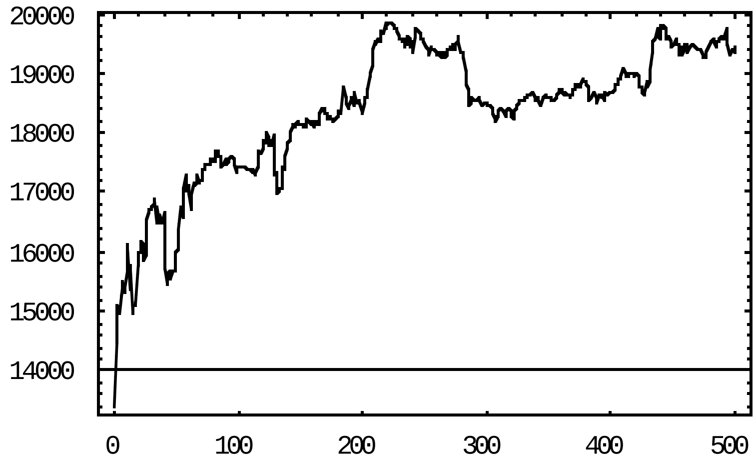
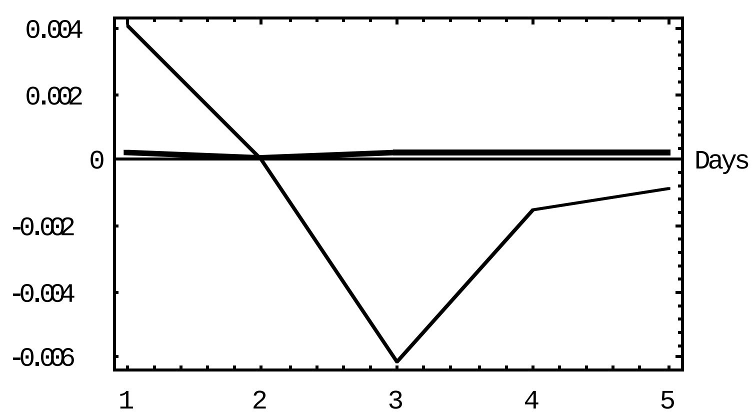


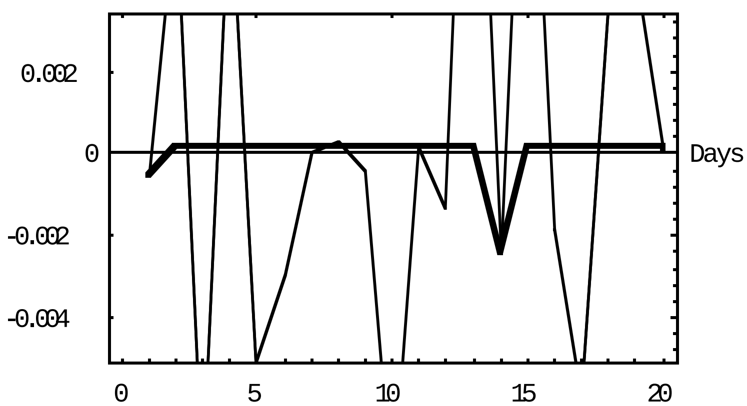
Figure 1f



It must be stated that the positive return on this simulation was achieved by keeping the system out of the market on all but one day. In any run, this could either be because the technical rule selected specified holding cash or because the confirmation process caused the system to default to cash. This run encountered a mixture of the two. Most importantly, however, the correct decision resulted.

In an attempt to reduce some of the problems caused by such a small sample size, another simulation was run to simulate 20 trading days. To accommodate the time constraint, only 20 generations were run each day. The crossover rate was again set to 0.50 and the mutation rate to 0.001. System performance also saw fitnesses improve from the 10,000 range to the 17,000 range. Figure 2 illustrates the return performance of the GTS versus the unmanaged market. The return of the GTS was 0.0373% (standard deviation of 0.0593%) and the comparable market return was 1.0573% (standard deviation of 0.6044%). After applying the Henriksson-Merton regression, no significant evidence of market timing ability was found ($p = 0.66$). The GTS was correct 10 out of 20 days for an accuracy rating of 50.0%.

Figure 2



It is unfortunate that more significant tests could not be presented here. Certainly further research would have to be done before any statistical significance could be attached to these conclusions. Still, the GTS does show increases in fitness for the whole system which indicates that the rules are improving and evolving over time. The time allowed in the system for realtime operation should be significant to obtain more powerful results over greater time periods. Running 2,000 to 3,000 generations per period should be sufficient to obtain valuable trading rules.

It should also be noted that the system saves the results of each generation (*e.g.*, the crossover point, the fitness, etc.) as well as the rule selected each day. Further, detailed information into the output of the confirmation process is also available from the model, but not presented here.

VI. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

This paper has shown three things: capital markets are not efficient and can be exploited for profit by technical analysis, technical analysis has a valid theoretical foundation and a testable formal theory, evolutionary computation represents a uniquely powerful way to test the theory presented here.

As opposed to previous attempts (most notably Allen and Karjalainen, 1993), this paper provides a general framework for evaluating the validity of technical analysis. Using the framework presented and the model written in *Mathematica* specifically for this application, legitimate testing of the efficient markets hypothesis with regard to the profitability of technical trading may be conducted. Unfortunately, there was insufficient data available for this paper to cover statistically significant tests of the hypothesis or of the theory of technical trading.

The controversy is far from settled. Although the theoretical framework and testing model presented here represent a paradigm shift in equilibrium analysis, many questions remain. In further research, a greater population of technical rules should be considered. It should be clear that the eight rules used in this model represent but an infinitesimally small subset of the number of existing technical trading rules. In this light, the eventual incorporation of a genetic procedure for searching for new rules (*e.g.*, Allen and Karjalainen, 1993) would be extremely valuable. To have the ability to exploit existing, theoretically sound rules for trading while searching for potentially useful new rules would greatly enhance the capacity for self-adjustment in the model.

With results arrived at through a more thorough running of the system, it would seem necessary to consider the possible utility of a scaling function for the current measure of fitness. While our current system has not indicated such a need, if the system appeared to achieve premature homogeneity, then such an adjustment would seem appropriate. In addition, another elementary representational consideration arises from the current methods of crossover and mutation. By allowing the all-zero prognosticatory rules located at potentially different positions between crossing strings to act as normally weighted rules, a systematic bias towards an excessive number of zeros for confirmatory rules crossed with the prognosticatory rules would seem inevitable. Future models should address this concern.

While the current model assumes that the investor is risk averse and therefore sets the default plan of action to hold cash, it would be enlightening to observe the model with the default set to buy. This new default would be indicative of a risk-seeking individual. In comparing the two results, one must keep in mind that the "buy" default is bound to have a higher level of risk since it would lead to greater market exposure. Higher returns from this method would need to outweigh the increased volatility implicit in such an assumption.

Further, an increase in the modularity of design would allow this model to accommodate different market structures and perhaps different assets (*e.g.*, commodities, currencies, fixed income securities, etc.). Other design modifications would include increased statistical testing facilities and greater use of visual analysis. Many trading rules consist of visually recognizing patterns in time series that ordinary time series analysis finds difficult to detect. Many of these are subjective, but many simply involve nonlinearities that conventional analysis cannot detect (see Scheinkman and LeBaron, 1989; Casdagli and Eubank, eds., 1992).

Finally, it would be advantageous for traders to analyze multiasset portfolios with this methodology. This would involve the incorporation of asset allocation analysis and multimarket microstructure considerations. The asset allocation decision is another problem area for capital market theory as there is no one formula for the proper allocation of investment capital. Employing utility theory in this manner produces highly subjective and often questionable allocation results. Thus, a means by which the allocation decision could be evolved along with the security choice decision would be very useful.

Nonetheless, this paper makes substantial inroads on addressing the lack of a formal theory of technical analysis and a valid test of such a theory. There is much insight to be gained from evolutionary computation as applied to such complex phenomena as optimal trading processes. The results presented here belie the power of the adaptive methodology in more detailed applications.

APPENDIX

A1. Introduction

There is a great deal of conflict among researchers as to the intellectual validity of technical analysis. However, there can be little conflict with regard to the empirical results of research into capital market efficiency. Despite such persistent skeptics' claims (*e.g.*, Malkiel, 1973) that the markets are efficient, empirical studies have shown repeatedly that they are not (Vaga, 1990; DeBondt and Thaler, 1991a; Peters, 1991; Allen and Karjalainen, 1993; Hawawini and Keim, 1994; Peters, 1994; Vaga, 1994; Pennar, 1995; Rode, 1995b). Peters (1991) addresses the issue of inefficiency in currency, commodity, and interest rate markets, Hawawini and Keim (1994) provide a comprehensive econometric analysis of international equity market inefficiencies, DeBondt and Thaler (1991b) discuss institutional constraints as theories of quasirationality, and Rode (1995a) reviews the decision-theoretic literature on capital market efficiency.

Under the weight of such a massive body of evidence, one must conclude at the very least that markets are not *perfectly* efficient. It is important to note, however, that there is also a conflict as to the precise nature of efficiency. Further, since capital market efficiency is almost always discussed in the context of investor rationality, one must question what it means to be rational as well. A large body of literature in the decision sciences draws several conclusions. Human decision makers clearly do not follow the decision processes discussed by economists and presumed in theoretical models (Rode, 1995a). Some of the most compelling work has been done by Nobel laureate Herbert Simon (Newell and Simon, 1972; Simon, *et al*., 1987) and involves a detailed analysis of where the theoretical process model breaks down in actual human thought. A fundamentally important conclusion emerges: investors are constrained to processes feasible given the task environment. The task environment places two constraints on investors: time and memory.

It is easy to see that investors who are unconstrained by cognitive limits on memory capacity and are not pressured by time could make optimal decisions. But humans are substantially limited in those capacities. The predominant decision environment for humans is one in which decisions must be made with incomplete and even unknown information and under time pressure. In these environments, humans use heuristics which they adapt over time to make reasonably good decisions in complex problem spaces. Rule-based thought and inductive logics are incomplete, and thus are confined to suboptimality, yet, it many situations, the complex evolution of decision processes according to simplifying heuristics produces significantly good results.

It is in that light that this paper addresses quantitative technical analysis. Technical trading rules can be viewed as simplifying heuristics that investors use as substitutes for further analysis. This reasonable enough conclusion also implies that markets are not efficient. Clearly, if investors are using suboptimal rules, then markets will return suboptimal results. The suboptimality presents itself in terms of investors "leaving money on the table." Investors, for whatever reason, are not obtaining maximum efficiency from the markets, thus, there is money to be made by understanding *how* investors trade suboptimally.

The foundations of modern portfolio theory and the highly regarded Capital Asset Pricing Model (CAPM) (Sharpe,1964; Lintner, 1965; Mossin, 1966) and Arbitrage Pricing Theory (APT) (Ross, 1976) appear to be less compelling than they once were. If markets are in fact necessarily inefficient, then the CAPM and APT are relegated to special case status - they are appropriate *only* if investors behave with perfect rationality. Only now are researchers beginning to make accommodations for actual human investors in asset pricing models (*e.g.*, Shefrin and Statman, 1994). Still, much work remains and little work has been done on the validity of technical trading rules. There is an enormous stigma. Academic research still looks on technical analysis with condescension because it is widely thought to lack a sound theoretical premise. Because of this, it is important to recognize that the theoretical premise of technical analysis lies in the fact that real world human decision processes *actually* do use heuristic rules, such as technical trading rules, to make investment decisions and that humans are *rationally* necessitated to do so because of constraints imposed by the task environment. Markets remain amazing institutions not because they eliminate inefficiency, but because they function *in spite* of it (Rode, 1995a).

This paper briefly outlines the theory underlying technical analysis and introduces many of the basic tools used in quantitative technical analysis. Section II introduces the theory of rule-based investment management. Section III details the tools of technical analysis. Section IV concludes the paper.

A2. Rationality and Rule-Based Investment Management

The object of technical analysis is to predict a complex time series with one which is easier to calculate and forecast. This is exactly the essence of simplifying heuristic behavior: substitution of the less complex for the intractable. Still, identifying the goal does little to provide insight on the process. Five factors are identified as the principal reasons why technical analysis works (Sears and Trennepohl, 1993):

1. The price of a security is determined solely by its supply and demand.
2. Prices tend to move in trends that persist for an appreciable time.
3. Changes in trends are caused by changes in supply and demand.
4. The patterns or trends tend to repeat themselves over time.
5. Supply and demand is governed by both rational and irrational factors.

What is most obvious in these factors is the presence of persistent, similar behavior. In earlier attempts to compensate for the evidence of market efficiency, it was proposed that such irregularities would disappear in the aggregate as one irrational act canceled out another. It appears today, however, that the interaction among various quasirational agents does not eliminate inefficiency. In fact, it may serve to *sustain* inefficiencies and *promote* further complexity.

Rule-based investing represents an attractive course of action because of the simplicity of its approach. Most investors do not have the time or the knowledge to apply advanced econometric techniques such as ARIMA analysis or GARCH modeling to analyze investments. Nor do they have the theoretical backgrounds to draw conclusions about specific investments from macroeconomic news. When investors consider the purchase of a security, have they fully analyzed every alternative investment available and extensively analyzed their own utility functions for risk tolerance? It is doubtful at best and most likely demonstrably no. It is simply unreasonable given the time pressure most investment decisions are made under. There are substantial opportunity costs in not investing promptly. For example, while an investor is contemplating which high technology stock to buy using multiattribute utility theory, his investment capital is sitting in a bank account earning the riskless interest rate. There is a substantial opportunity cost to the gathering of complete information. Thus, it is often better for investors to supplement heuristic rules for formal analysis. Any potential loss of return because of suboptimal analysis may, in fact, be compensated for by the opportunity cost *not lost* by acting promptly.

It is further important to realize that technical analysis and investment rules need not be constant over time. Just as situations can change, so must rules adapt with them. Investors will routinely observe a large “portfolio” of rules when using rule-based analysis. There is no one technical indicator that consistently returns correct forecasts. Investors look for confirmation among the rule portfolio as to the general sentiment of the market. It is easy to see, therefore, the valuable role which expert systems may play in such an environment. The simplest of expert systems would present investors with the output of a portfolio of rules as well as information on their past performance. Using this information, investors could subjectively weight the forecasts of the rule portfolio. More complex expert systems could be built to analyze the interaction among the rules. Although it is an oversimplification, they are complex pattern-matchers.

The rule portfolio is important because of the fact the no one rule is overarchingly correct. Pattern-matching rule-based expert systems can be used to extract with some accuracy the rule portfolio weightings used by the population of heuristic investors as a whole. Given a known portfolio of rules from which to choose and a long enough time series, algorithms can be created to determine the weights on the rules of the underlying portfolio and how they change over time. With such knowledge, results similar to the following can be created: When α happens while β is dropping, investors will depend more heavily on rule γ to make a decision - rule γ's output is $f(\gamma)$. With an informed forecast of the rules to be used and the rules' output, portfolio managers can adjust their portfolios accordingly.

A3. The Technical Analyst's Toolbox

Models for technical analysis can be placed in three categories: cyclical indicators, trade volume indicators, and value indicators. This paper will not address fundamental analysis (roughly defined as the estimation of the "true" value of a firm based on financial reporting) or chart analysis (identification of visually familiar patterns in price histories). This paper will most likely not address the entire portfolio of rules of any given type. Each trader is likely to have his or her own versions of certain rules and create new rules as he or she sees fit. Rules are also altered to make allowances for different trading environments (equities, commodities, currencies, etc.). This section will outline the most basic formulations of the most common rules and is very loosely based on Thomsett (1989), Gietzen (1992), and Sears and Trennepohl (1993).

Cyclical Indicators

Cyclical indicators are concerned with finding sustainable patterns of price movement in the market. These tools are primarily used to determine whether the current market is in an *up trend* or a *down trend*. The primary tools used for cyclical analysis are moving averages (simple, weighted, and exponential) and detrending analysis (cyclical curve-fitting). Moving averages can be represented in a variety of forms, but all have essentially the same effect: to smooth the data. By using moving averages, statistical outliers are omitted from confusing the presence of actual trends in the market. It is viewed as bullish when shorter term moving averages "break through" or exceed the value of longer term moving averages. Similarly, it is seen as bearish when the shorter averages fall below the longer averages. Figure One illustrates various lengths of simple moving averages. The general formulas for moving averages are:

Simple Moving Average (Π_t is the period *t* price)

$$\Pi_t = \frac{1}{n} \sum_{i=t-n}^{t} \Pi_i$$

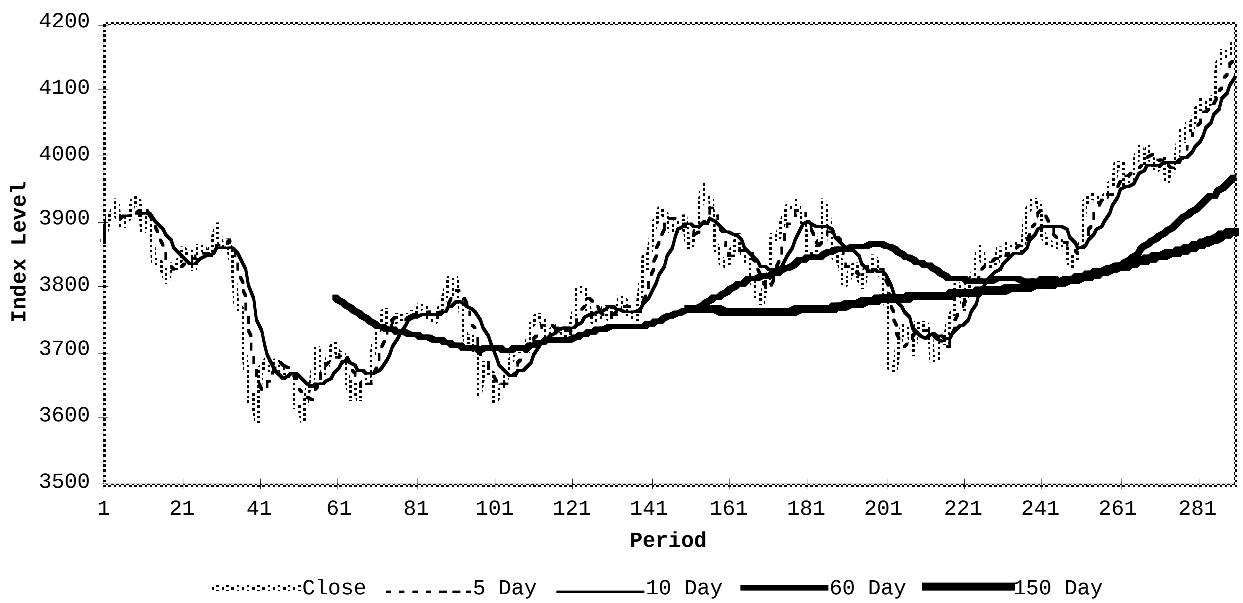
Weighted Moving Average ($\omega_i \in [0,1] \;\; \forall\, i, \; \sum_i \omega_i = 1$ is the period *t* weight)

$$\Pi_t = \frac{1}{n} \sum_{i=t-n}^{t} \omega_i \Pi_i$$

Exponential Smoothing (α is the smoothing factor)

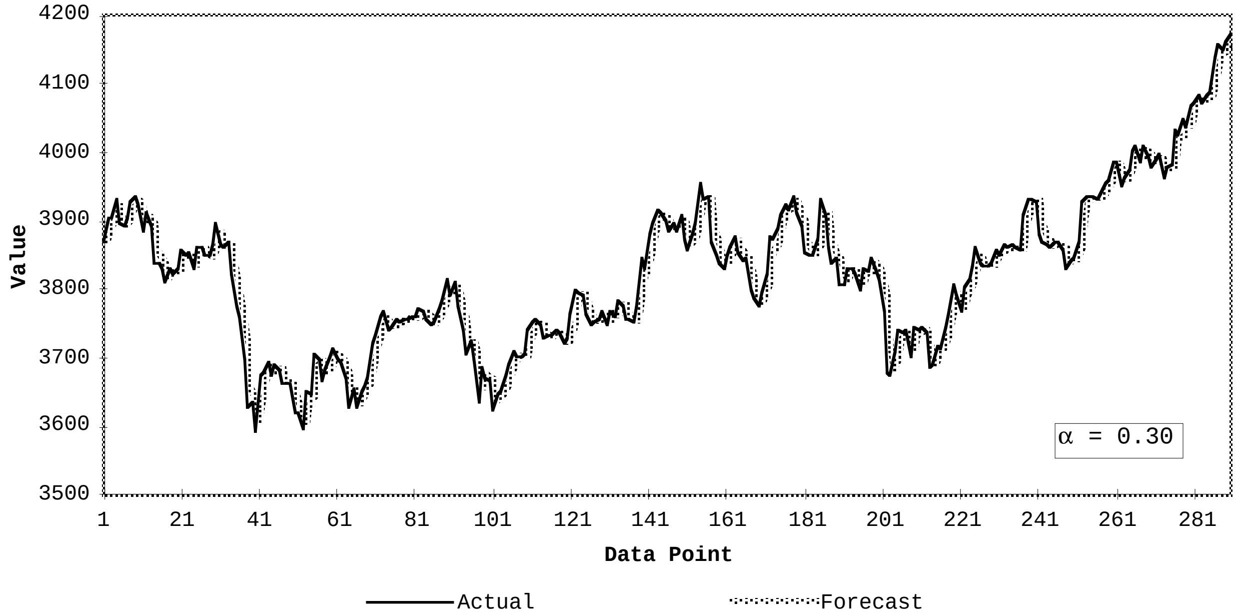
$$\Pi_t^{\exp} = \alpha \Pi_t^{\exp} + (1-\alpha)\, \Pi_{t-1}^{\exp}$$

Figure One



The exponential smoothing method is particularly attractive for behavioral analysts because it more heavily weights recent information, which is compatible with most quantitative learning models. It is also simple to calculate because of its recursive form. An exponentially smoothed time series is presented in Figure Two.

Figure Two



One could also make the assumption that market cycles can be represented by a regression on trigonometric sine waves. A note of caution here: as with most curve-fitting techniques, the amplitude or frequency of the waves can and does change over time. Thus, out-of-sample, the fitted equation may be useless. Still, for very short-term forecasting, it has some value. Accordingly, the equations are presented below. $\bar{\Pi}_m$ is the centered average over the period of the longest cycle, T_m. A_n is the cycle amplitude. $\Theta_n = 2\pi\left(\frac{\tau_n}{T_n} - 1\right)$ is the time point of each cycle, and τ_n is the cycle period.

$$\Pi = \bar{\Pi}_m + \sum_{n=1}^{m-1} A_n \sin\Theta_n$$

Lastly, a model called "slow stochastics" is used. Slow stochastics is based on the theory that prices tend to close near the top of daily ranges during uptrends and near the bottom during downtrends. The levels of 30 and 70 are (arbitrarily) set as the extremities. If the indicator diverges while in extreme conditions, a buy (or sell) signal is generated accordingly. This divergence is measured by the *KD Oscillator*.

$$C(d)_t \equiv \Pi_t^C - \inf\left\{\Pi_{t-k}^L\right\}_{k=1}^d$$

$$\mathbb{H}_t \equiv H(d)_t - L(d)_t = \sup\left\{\Pi_{t-k}^H\right\}_{k=1}^d - \inf\left\{\Pi_{t-k}^L\right\}_{k=1}^d$$

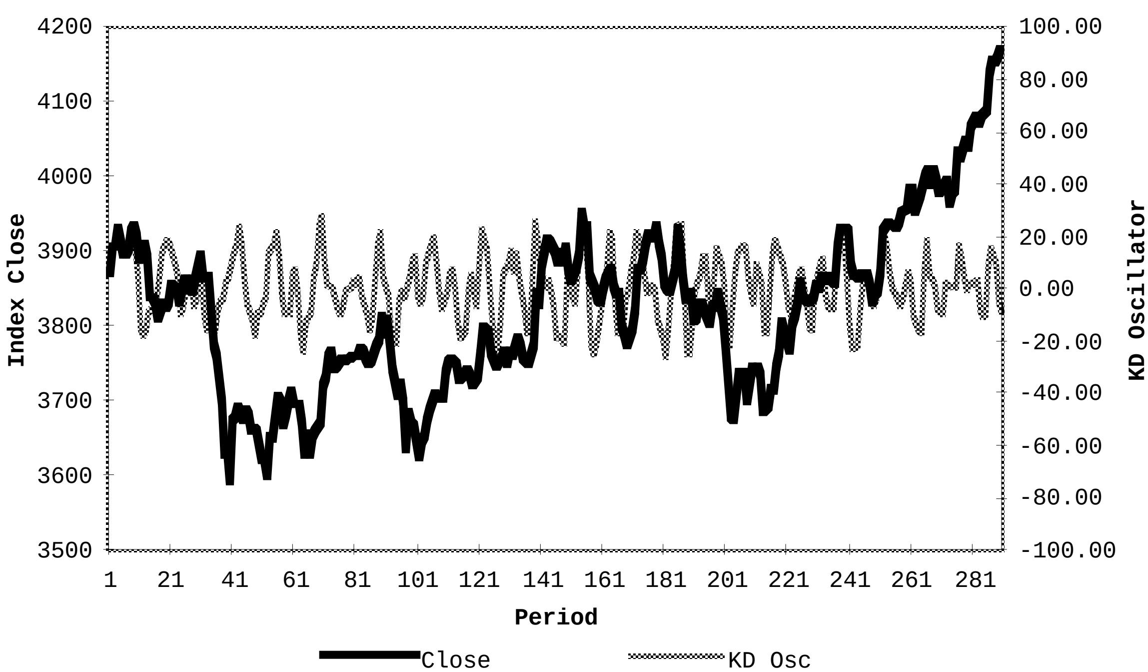
$$K_t = \frac{100 \cdot \sum_{i=t-3}^{t} C(d)_i}{\sum_{i=t-3}^{t} \mathbb{H}_i}$$

$$D_t = \frac{1}{3}\sum_{i=t-3}^{t} K_i$$

$$KD_t = K_t - D_t$$

$C(d)_t$ represents the current closing value, C, minus the minimum of the low for the previous d periods. $\mathbb{H}_t$ represents the difference between the maximum of the past d highs (H) an*d* the minimum of the past d lows (L). The remaining equations serve to normalize the divergence statistics (K and D) to between 0 and 100 so that the KD Oscillator can fluctuate about zero. The KD Oscillator, KD_t, is illustrated in Figure Three.

Figure Three



Each of these indicators has value for a particular application. Moving averages of all types are widely used to obtain clearer pictures of trends in time series. Sinusoidal regression and regression of other transcendental functions can be used as a simple way to extract short-term price tendencies using the natural form historically found in market data. Slow stochastics represent a measure of divergence that can be used to signal trend reversals. These and other measures of trend analysis represent the most frequently used tools in technical analysis.

Trade Volume Indicators

Clues as to how the market will react and investors behave can also be found in data which analyzes the *intensity* and *direction* of trading activity. Two measures of such will be addressed here: the Arms index (named after Richard Arms) and the Advance-Decline line.

The Arms index is a popular measure of very short term market sentiment and incorporates both the number of advancing and declining issues for a particular index and the corresponding volumes for those issues. Let A represent the index value and N^+ equal the number of advancing issues, N^- the number of declining issues. V_A and V_D will, respectively, represent the volume of the advancing and declining issues. Therefore, the Arms index, A is

$$A_t = \frac{N^+ V_D}{N^- V_A}$$

A value of 1.0 is neutral with values less than 1.0 bullish and greater than 1.0 bearish. Values of about 0.75 or less indicates strength and values over 1.0 indicate weakness. Further, values less than 0.5 soon after the start of trading give a high probability of a strong rally. Used as a longer term indicator, the 10-day moving average Arms index can indicate overbought (bearish) conditions if it reaches 0.8 or below and oversold (bullish) conditions is it reaches 1.2 or greater.

There are several measures of the Advance-Decline line. This paper will outline three of them: exponentially smoothed Advance-Decline, Overbought/Oversold, and Plurality. Each of these indicators is used as a measure of *market breadth*. That is, the degree to which investors are all behaving in a similar fashion. This may be an indicator of the "herd instinct" that may lead to nonequilibrium selling or buying of securities. In the following equations, the letters A, D, and U will be used to represents the number of issues advancing, declining, and remaining unchanged, respectively. V denotes a volume statistic.

Exponentially Smoothed Advance-Decline Index

$$N = 100 \cdot \frac{A - D}{A + D + U}$$

$$N_n^{\exp}(\alpha) = N_{n-1}^{\exp}(\alpha) + \alpha\left[N_n - N_{n-1}^{\exp}(\alpha)\right]$$

20-day Overbought/Oversold Index

$$OBOS = \frac{\sum_{i=t-t^\star}^{t} A_i}{\sum_{i=t-t^\star}^{t} D_i}$$

Plurality (φ) and the Advance/Decline (AD) Line

$$\varphi = \sum_t \left(A_t - D_t\right)$$

$$AD_t = AD_{t-1} + (A - D)_t$$

It is important to note that there are no hard and fast rules with respect to the use of trade volume indicators. Most often, they are used as confirming indicators to use with other, more specific indicators, to eliminate spurious correlations.

Value Indicators

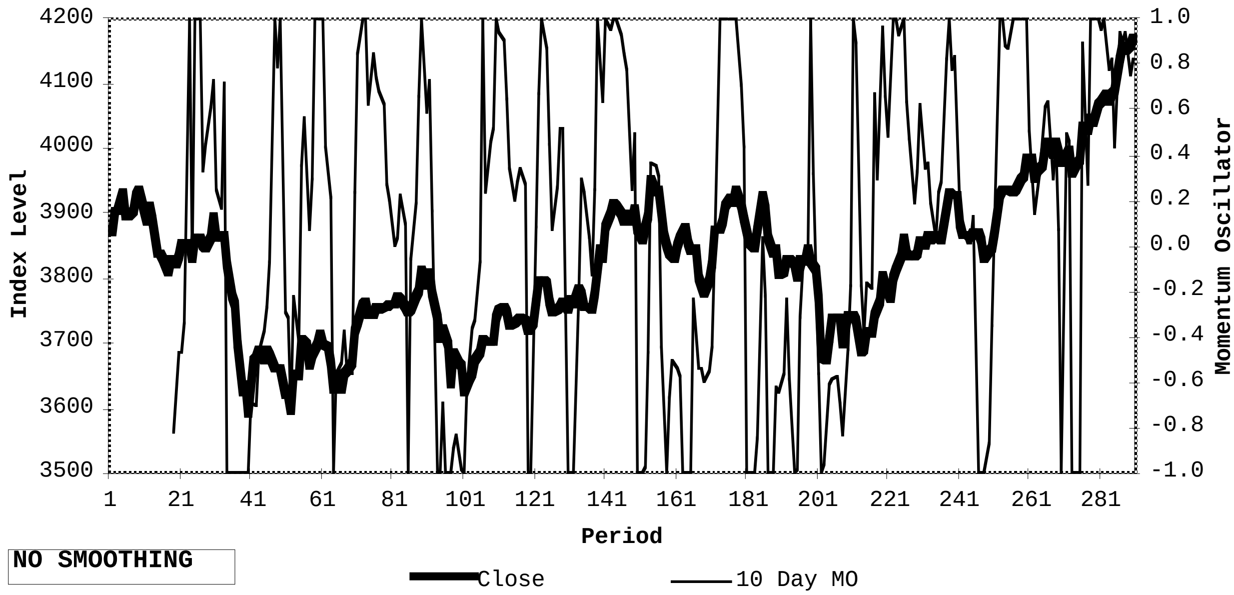
Value in this context nearly always refers to *relative* value. Measures which allude to this relative value concept are Momentum, Relative Strength, and Accumulation-Distribution.

Momentum indicators use the rate of change of the market to indicate overbought and oversold conditions. Overbought conditions are signaled by indicators in the upper ranges and the market is termed oversold in the lower ranges. The *momentum indicator* (*MI*) is used to calculate the *momentum oscillator* (*MO*). The formulas for both are given below and Figure Four illustrates the indicators. n is the duration of the indicator.

$$MI_t(n) = \Pi_t - \Pi_{t-n}$$

$$MO_t(n) = \begin{cases} \frac{MI_t(n)}{\sup\{MI_{t-k}(n)\}_{k=1}^{n}} & MI_t(n) > 0 \\ 0 & MI_t(n) > 0 \\ \frac{-MI_t(n)}{\inf\{MI_{t-k}(n)\}_{k=1}^{n}} & MI_t(n) > 0 \end{cases}$$

Figure Four



Relative Strength Indices (RSIs) are forms of momentum oscillators which smooth price changes through averaging and normalize the values to an index between 0 and 100. Movements above 70 are (arbitrarily) considered overbought, movements below 30 are (arbitrarily) considered oversold. Figure Five illustrates the 15-Day Relative Strength Index

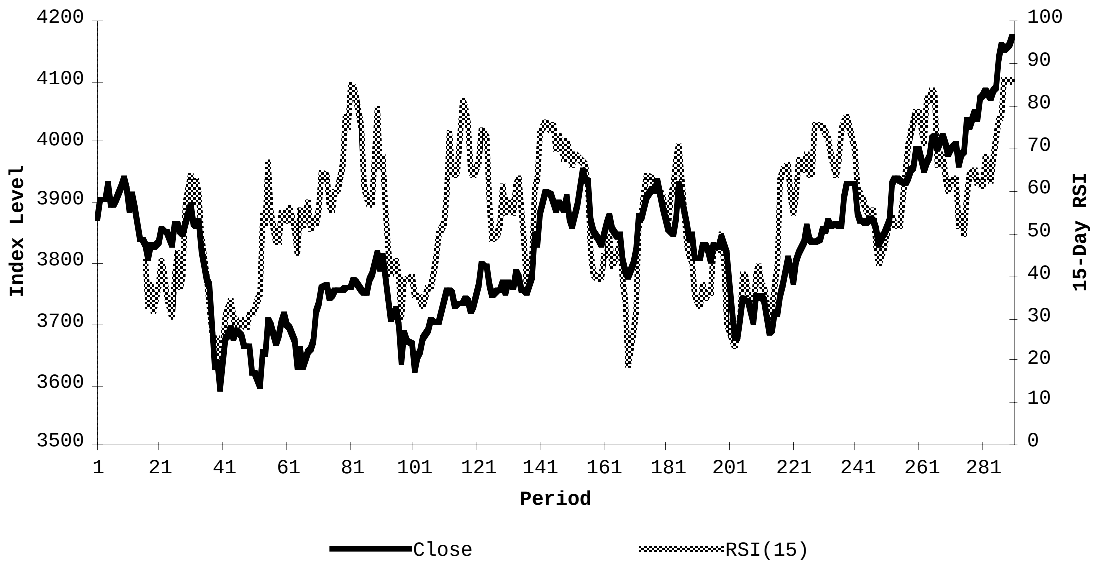
$$\Delta_t = \Pi_t - \Pi_{t-1}$$

$$RSI_t^u = \begin{cases} \Delta_t & \Delta_t > 0 \\ 0 & \Delta_t \le 0 \end{cases}$$

$$RSI_t^d = \begin{cases} \Delta_t & \Delta_t < 0 \\ 0 & \Delta_t \ge 0 \end{cases}$$

$$RSI_t^k = 100 - \left[\frac{100}{1 + \dfrac{\frac{1}{n}\sum_{i=t-k}^{t}\left(RSI_i^u\right)_i}{\frac{1}{n}\sum_{i=t-k}^{t}\left(RSI_i^d\right)_i}} \right]$$

Figure Five



The Accumulation-Distribution index is used to find divergence or turning points in trend lines. Because of its construction, it follows price closely and is seen to be bullish if within some time period *n*, a subsequent low is greater than a previous low and bearish if a subsequent top is less than a previous top. It represents the Accumulation-Distribution Index at time *t*.

$$I_t = \begin{cases} I_{t-1} + \left(\Pi_t^C - \Pi_t^L\right) & \Pi_t^C > \Pi_{t-1}^C \\ I_{t-1} + \left(\Pi_t^H - \Pi_t^C\right) & \Pi_t^C < \Pi_{t-1}^C \end{cases}$$

As one can see, a wide variety of statistics can be calculated in addition to these measures presented here. Additionally, many investors modify these rules by incorporating averages to smooth out the volatility in them. These modifications take many forms and serve to adapt the statistics to not only the market (by adjusting to local cycles) but to the desired investment goal (short term or long term trading) as well.

A4. Conclusion

The world of technical analysis is wide open. There are very few absolute rules with regard to the proper numerical analysis of data. It is doubtful if there will ever be the definitive rule portfolio. Still, empirical evidence has shown time and again that a carefully updated rule portfolio can allow investors to achieve not only excess returns (alpha gains) but also achieve significant reductions in risk over an unmanaged, “buy and hold” strategy.

The common complaint of traditional economists about technical analysis being a “theory without theorems” was indeed true and represented a life-or-death challenge for technical analysts. Fortunately, plausible and economically sound foundations for technical analysis have been proposed. Naturally, further analysis is necessary, but recent advancements in evolutionary economic methods and decision-theoretic process models provide significant support for the rule-based investment process and quantitative technical analysis.

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