$$\sigma_\rho^2 = \sigma^2/H + (V/H)^2 \cdot \frac{(1-\gamma)[1-(1-\gamma)^H][(1-\gamma)^{H+1} + \gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}]}{[(1-\gamma) + \gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}]^2}, \quad (44)$$

$$P(\rho \mid X_{-\infty}, \cdots, X_{t_I}t_I)$$

$$= \left[\frac{(1-\gamma)^{H+1} + \gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}}{(1-\gamma) + \gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}}\right]\frac{e^{-\rho^2/2(\sigma^2/H)}}{\sqrt{2\pi}(\sigma/\sqrt{H})}$$

$$+ \left[\frac{(1-\gamma)[1-(1-\gamma)^H]}{(1-\gamma) + \gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}}\right]\frac{e^{-(\rho-V/H)^2/2(\sigma^2/H)}}{\sqrt{2\pi}(\sigma/\sqrt{H})} \quad (45)$$

VI. Conclusions

Adherents of technical analysis claim that unusual profit can be achieved using only past security prices. Most academics believe that the securities markets are efficient enough to make this impossible. This paper has shown that past prices, when combined with other valuable information, can indeed be helpful in achieving unusual profit. However, it is the nonprice information that creates the opportunity. The past prices serve only to permit its efficient exploitation.

REFERENCES

1. Robert Ferguson. "Active Portfolio Management—How to Beat the Index Funds." *Financial Analysts Journal* 31 (May/June 1975) 63–72.
2. Jack L. Treynor and Fischer Black. "How to Use Security Analysis to Improve Portfolio Selection." *Journal of Business* 46 (January 1973) 66–86.

DISCUSSION

ERIC H. SORENSEN*: This paper by Jack Treynor and Robert Ferguson (TF) is an attempt to demonstrate the usefulness of knowing past price information in making investment decisions. The normative contribution of the paper is the development of a Bayesian probability estimate to assess using past price data whether or not the market has already incorporated some firm-specific information, which has been made available to the investor. The obvious implication is that if the market *has not* discovered the information, and this can be confirmed with past price data, then the holder of private information can act accordingly.

TF conclude: "This paper has shown that past prices, when combined with other valuable information, can indeed be helpful in achieving unusual profit. However, it is the nonprice information that creates the opportunity. The past prices serve only to permit its efficient exploitation."

I have considerable pragmatic compassion for the notion. I have worn my money manager hat enough to come to believe that the examination of past *and*

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current price data is valuable in the presence of some subjective opinions about the fundamentals of a firm. For example, other things equal, I prefer to buy my straw hats in the fall (typically after a systematic reduction in price). The difference between this hat-buying investment philosophy and that which emerges in TF is that I know it's fall by reviewing my calendar. In contrast, TF arrive at a probability that it is fall by reviewing the recent trend in straw hat prices.

Contrasting of these two approaches leads to one critical question: How reliable is my calendar? This question is not clarified in the paper. In fact, the question of information value is assumed to not be an issue because it is treated as exogenous and deterministic.

In the introduction, TF posit that there are two ingredients to successful usage of technical analysis: (i) past price data and (ii) valuable information of a firm-specific nature. By assuming a "value" associated with a prior information event, the model collapses to a problem of statistical measurement of past price movements.

This simplification in modeling is analogous to a case in which a mechanical engineer might analyze the economics of the internal combustion engine without knowledge of the relative prices of the necessary ingredients for combustion. Successful combustion requires air and petroleum. The assumption that the supply of petroleum is a free and exogenous good reduces the analysis to one of economic insignificance. Similarly, TF concentrate on past prices (free air), and consequently they lose some degree of richness in the problem by failing to incorporate the importance of valuable information (scarce petroleum).

The assumptions (explicit and implicit) in this paper pop up throughout the manuscript. A review of them adds clarity and puts the model in perspective.

Information Assumptions

1. Relevant information is firm-specific.
2. Information is well-defined in that it has a known value, and that all investors have the same value assessment.
3. A piece of information can be identified with a specific event, and there is no other related information which confounds the event-information linkage.
4. Events, and thus information, are infrequent, relative to the transmission time interval.
5. There is no false signalling; information has a known quality and is believable.

Behavior Attributes

1. The investor acts deterministically upon receipt of information.
2. The investor can calculate a number of priors, and conditional probabilities: α, γ, $P(t_M \mid t_I)$, $P(X - \infty, \cdots X_t \mid t_M t_I)$, $P(t_E \mid t_I)$, $P(t_E)$, and V.

Market Characteristics

1. There are two players: the investor and "the market."
2. The investor is a price taker, whereas the market makes prices.
3. All price changes are the result of information provided to "the market."
4. Most price changes result from small pieces of information, whereas the event associated with t_E is relatively large.
5. The market is "efficient," in the price reaction sense.

It is important for us to know which of these assumptions are most important and/or require the largest measures of faith. One wonders how large V (the "value" of the information) must be, relative to the return distribution, in order to make the problem an interesting one. The type of distribution may bear upon the answer.

One feels the urge to expand the scope of the model. For example, the "efficient market" assumption describes a price reaction as being instantaneous after the market's receipt of information, t_M. However, the opportunity of $t_I < t_M$ describes an inefficient market in the expected return sense. Additional players with access to $t_I < t_M$ present a reduction in "price reaction efficiency" traded-off for an increase in "informational efficiency."

Another expansion to the model could be a detailed discussion of the likelihood that the information received by the investor is valid. If t_E is known to have occurred, then $P(t_M \mid t_I)$ is either zero or one, depending upon the magnitude of V. In the example, V is "large" (i.e., it is 2 standard deviations beyond the mean). If t_E has occurred, and if past prices are abnormal, then $P(t_M \mid t_I) = 100\%$. On the other hand, if prices are not abnormal, then $P(t_M \mid t_I) = 0\%$. Moreover, if V is not a large event, then $P(t_M \mid t_I)$ can make no use of prior prices, and the investor is totally dependent upon his priors.

Given the assumptions set forth above, the TF paper may have restricted applicability. Nevertheless, within the confines of the TF model, the crux is this: Does the trader's willingness and ability to estimate α, γ, and all the $P(\cdots)$'s of the model dominate his seat-of-the-pants skill and intuition. The answer is, of course, itself highly subjective. Most traders spend their resources trying to discover the petroleum rather than measuring the air. If this is true, the problem solved in TF is not an important one. Economics aside, even if we do confine our analysis to a Bayesian measurement of the air, we may be disappointed to find that the game is worth winning, but no fun playing.