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In Defense of Technical Analysis

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ABSTRACT

Many investors occasionally receive what they believe to be nonpublic information about a security. Others feel that by applying superior analytical skills to public information, they are able to arrive at valuable insights that are not generally appreciated. In either case, there is a substantial opportunity for profit if the investor is correct. The investor must be correct on two counts. First, the estimate of the worth of the information must be reasonably accurate in terms of its impact on the price of the stock, and second, the investor must make a realistic assessment of the likelihood that the market already has received the information or insight in question. This paper is concerned only with the latter problem. The probability distribution of the date on which the market receives information already in the hands of the investor is calculated for a simple model of information propagation. It is then shown how this probability distribution can be brought to bear on the management of a portfolio.

SUPPOSE AN EVENT OCCURS which has an impact on the business prospects of a single company. Any investor who is informed of this event will revise the estimate of the value of the company's stock and hence its return/risk characteristics. Assuming the investor felt he or she owned a suitable amount of the stock just before receiving the information, the investor's reassessment of its return/risk characteristics in relation to those of other securities will lead him or her to conclude that he or she owns too little or too much now. As a result, the investor will either purchase or sell shares. Since the investor can accomplish this only by enticing other investors who have not received the information to act in a complementary manner, which requires a contrary reassessment of the stock's return/risk characteristics (but not value) on their part, the impact on the stock's price will have to be a change in the direction of the first investor's reassessed value. But only when all investors have received the information will the stock's price reach the revised value.[1] Furthermore, the price change produced by the spread of this information will be a unique return in the Treynor-Black sense[1,2] since no other company's prospects are involved.[2]

If events of this sort were few and far between, then a plot of the cumulative daily unique return of a stock corresponding to a particular event might look like Figure 1A. Implicit in that drawing is the assumption that it takes several days for information to propagate throughout the investment community. In a market where information spread more rapidly, Figure 1B would be more accurate. And

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[1] Investors are assumed to have homogeneous beliefs when they have identical information. This assumption is not necessary, but makes the exposition easier.

[2] The argument can be generalized to information other than that pertaining to a single company.

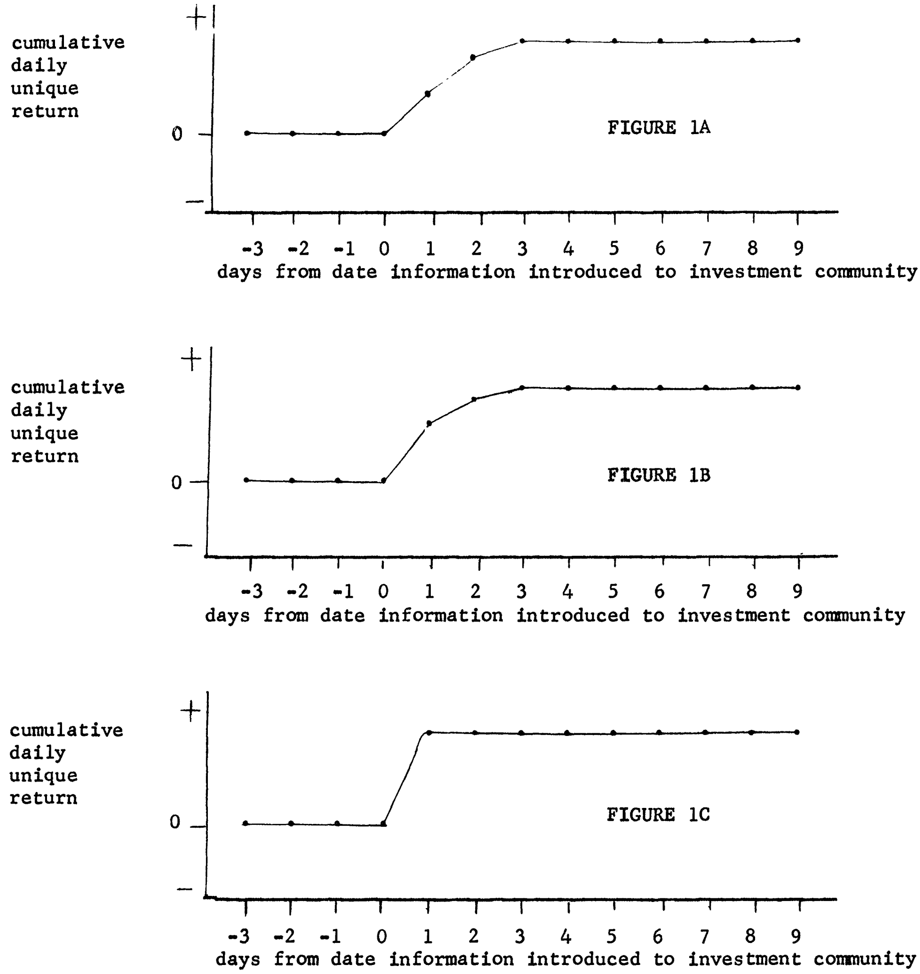

Figure 1

in a highly efficient market, Figure 1C would be the case.[3] In terms of daily unique returns, these patterns translate to the patterns in Figure 2, A–C, respectively.

Suppose the market is efficient.[4] Then the patterns in Figures 1C and 2C are the relevant ones, and a stock's price changes from its initial value to its revised value the same day information is introduced to the investment community. But this date may be sometime after the event which the information refers to. As a result, some few privileged investors may obtain the information prior to its

[3] The propagation of information can be likened to the spread of a disease. This view leads to a characteristic pattern of cumulative unique return where the rapidity of approach to the new equilibrium price depends on a single parameter in much the same fashion that a nuclear decay rate can be expressed in terms of a half-life. This parameter can be used as a definition of market efficiency.

[4] A reasonable assumption in light of the evidence.

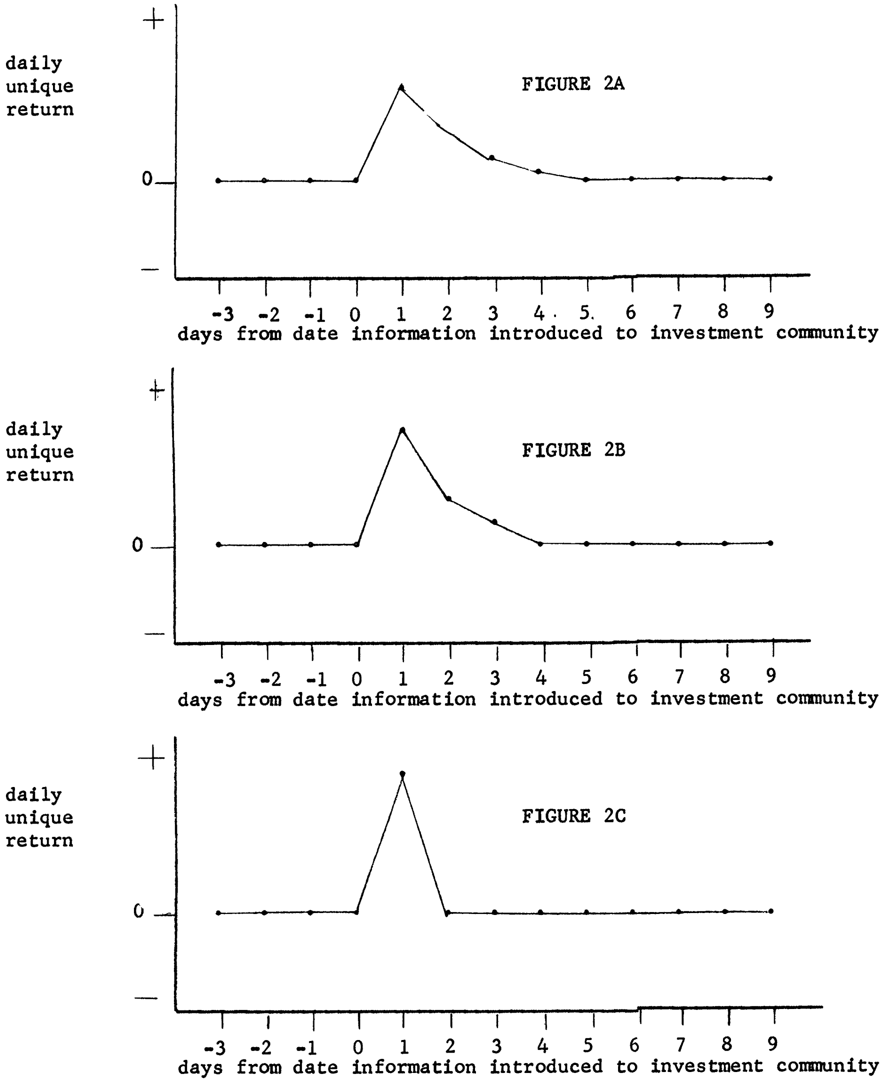

Figure 2

introduction to the investment community. Thinking of the market as a single monolithic entity, then if the investor receives the information before the market does and takes an appropriate position, he or she can expect a profit on the day the market receives the information. If, on the other hand, the investor takes his position and it turns out that the market received the information first, then no profit will be forthcoming and he or she will lose by the amount of his transaction cost and suffer an additional opportunity loss. If the investor takes no position, then he or she suffers an opportunity loss if the market has yet to receive the information and suffers no loss if the market already has the information.

Suppose that the typical time from the date of an event, t_E, to the date its occurrence is communicated to the market, t_M, is very short compared to the typical time between events. Suppose also that the same is true of the time it takes for the investor to learn of an event. Then there is a simple solution to this dilemma. *Assume* that whenever the investor receives information about a security, at t_I, he or she plots its past unique returns $(X_{-\infty}, \cdots, X_{t_I})$. If the market already has received the information, there will be a tell-tale relatively recent spike of the proper size, as in Figure 2C. Any other spikes will be old. In this case, the investor will know that this information is valueless and will not establish a position. If there is no relatively recent spike of the proper size, the investor will know that he or she received the information first and will establish an appropriate position.

The above procedure provides the investor with the probability that t_M exceeds t_I given the security's past unique returns, $P(t_M > t_I \mid X_{-\infty}, \cdots, X_{t_I})$. It is this probability that makes possible an intelligent decision concerning the establishment of a position. And under the assumption that $(t_M - t_E)$ and $(t_I - t_E)$ are typically very short compared with the time between events, $(t_{E_i} - t_{E_{i-1}})$, the investor will obtain either 0 or 1 for this probability. But if the time between events typically is not long compared with $(t_M - t_E)$ and $(t_I - t_E)$, then it will not generally be clear when the market received the information. A plot of past unique returns will have many recent spikes corresponding to other events and a particular day's spike may correspond to more than one event. Assume the situation with respect to the event under consideration is as shown in Figure 3A and that the impact of other events is as shown in Figure 3B. The net result will be the pattern shown in Figure 3C. As can be seen, no obvious evidence of the spike in Figure 3A remains, and $P(t_M > t_I \mid X_{-\infty}, \cdots, X_{t_I})$ is no longer easy to obtain. In fact, the unique returns in Figure 3C for several dates other than t_M are closer to the right size than the one for t_M.

One way of attacking this problem is to use knowledge of the process by which information reaches the market and the investor to suggest which dates are the most likely ones. If certain dates are much more likely a priori candidates for t_M than others, then this should partially offset the fact that some of these other dates have unique returns more commensurate with the information. Suppose, e.g., that it is known that information concerning an event always reaches the market within six days and always reaches the investor within eight days. Then in examining past unique returns, the investor can ignore all but the most recent eight days, even if some earlier days have precisely the right size unique return and none of the most recent eight days does.

In the above example, knowledge of both t_I and the process by which information reached the market, and the investor provided information about t_M. Generally, such knowledge will permit the investor to calculate the probability of t_M given t_I, $P(t_M \mid t_I)$. If the investor also has knowledge of the process by which the unique returns caused by events other than the one under consideration are generated, he or she can then calculate the probability of a particular past history of unique returns given t_M and t_I, $P(X_{-\infty}, \cdots, X_{t_I}/t_M t_I)$. Combining this with $P(t_M \mid t_I)$, the investor can obtain the probability of t_M given $(X_{-\infty}, \cdots, X_{t_I})$ and t_I, $P(t_M \mid X_{-\infty}, \cdots, X_{t_I} t_I)$. This enables him or her to calculate $P(t_M > t_I \mid X_{-\infty}, \cdots, X_{t_I} t_I)$, which is what the investor requires to make his or her decision about establishing a position.

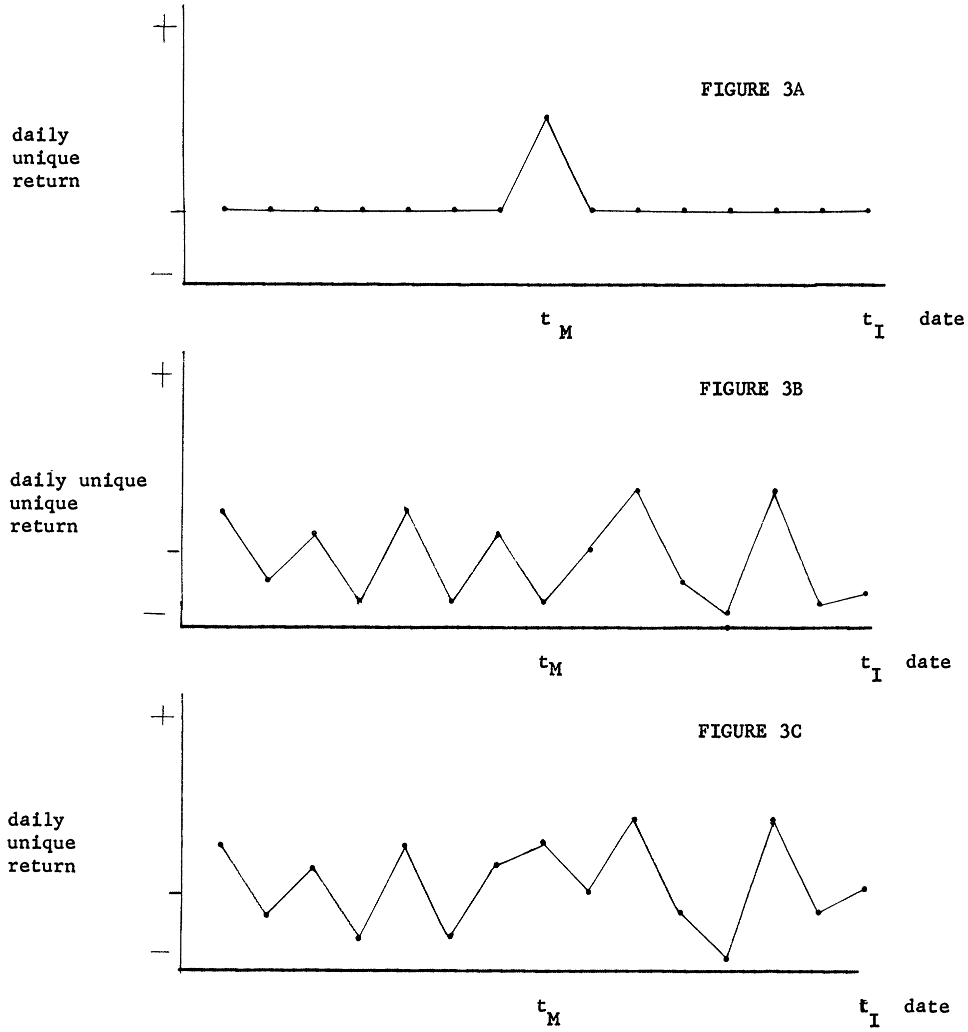

Figure 3

A more detailed and formal discussion of these issues follows.

I. A Recapitulation of the Problem

Consider an investor who receives what he or she believes to be nonpublic information about a security at time t_I. To use this information effectively, the investor must be concerned with:

1. The value of the information in terms of its impact on the price of the stock.
2. The process by which information is transmitted to the market and to the investor.

3. The probability that the market will receive (or has received) the information at a particular time, t_M.
4. A portfolio strategy which permits capitalizing on the information.

Each of these four issues is examined in turn in the remainder of the paper.

II. The Value of the Information

Probably, the simplest way to characterize the value of the information is to think of it as a specific dollar amount. A more sophisticated and more realistic view is to think of the amount as jointly determined by the communication under consideration and other events, which have occurred or will occur at various times, that have a direct or indirect impact on the security.[5] In this case, the value of the information will be a function of time, and the size of its impact on the price of the stock will depend on the specific date, t_M, on which the market receives the information. This paper takes a simple approach. The effect of the information is assumed to be a change in the price of the security by a multiplicative factor of e^V, where V is defined to be the value of the information. Roughly, speaking, the information is assumed to change the price of the stock, in the period ended at t_M, by 100%. In what follows, V is assumed known. Incorporating a probabilistic specification is straightforward.

III. The Propagation of Information

The process by which information is assumed to be transmitted to the market and to the investor is represented schematically in Figure 4. In that figure, there are four nodes labeled S_1, S_2, S_3, and S_4. Each node represents a state of nature. Specifically:

S_1 = the state in which neither the market or the investor have received the information;
S_2 = the state in which the investor has received the information but the market has not;
S_3 = the state in which the market has received the information but the investor has not; and
S_4 = the state in which both the market and the investor have received the information.

At the time the event occurs, t_E, the system is in state S_1. In each succeeding period, it may transition to any of the other three states. Suppose that the process by which a transition occurs is akin to flipping two biased coins, one for the investor and one for the market, where each double flip represents one period and tails denotes receipt of the information. Let γ denote the probability of tails

[5] Suppose the investor is told that a company has just received a contract from the government. For the moment, the investor would have to guess about its size. Suppose a few days later this investor hears that the government has just let a $100 million contract for the same product this company makes. Now the investor knows the size. The value of the information in the first communication could be viewed as dependent on the second communication, or vice versa.

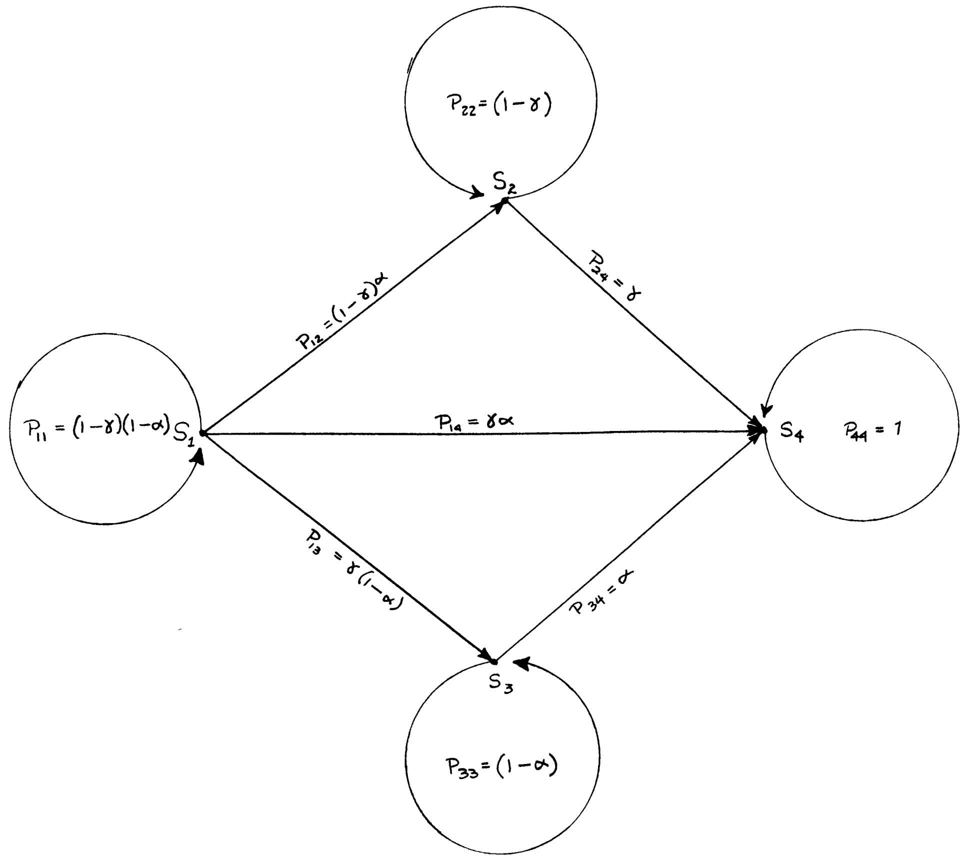

Figure 4

for the market's coin and α denote the probability of tails for the investor's coin. Then:

γ = the probability that the market will receive the information in the next period if it does not have it now; and

α = the probability that the investor will receive the information in the next period if he does not have it now.

After the event occurs, the system may remain in state S_1 for some time. The probability, P_{11}, that it will be in state S_1 in the next period if it is now is $(1-\gamma)(1-\alpha)$. The probability, P_{12}, that it will transition to state S_2 in the next period if it is in state S_1 now is $(1-\gamma)\alpha$. Denoting:

P_{ij} = the probability that the system will be in state S_j in the next period if it is in state S_i now.

Then a similar calculation for each possible transition yields the results shown in Table I. In Figure 4, the circular arrows enclosing probabilities denote transitions corresponding to remaining in the same state for one more period. The straight arrows between states, with their probabilities, denote transitions from

Table I
Transition Probabilities

		To			
		S_1	S_2	S_3	S_4
From	S_1	$(1-\gamma)(1-\alpha)$	$(1-\gamma)\alpha$	$\gamma(1-\alpha)$	$\gamma\alpha$
	S_2	0	$(1-\gamma)$	0	γ
	S_3	0	0	$(1-\alpha)$	α
	S_4	0	0	0	1

one state to another. All possible transitions and their respective probabilities are shown. This is a Markov process whose transition matrix is Table I.

Once the investor has been communicated with, he or she will want to calculate the probability distribution of t_M. In doing so, the investor should make use of the information about t_M contained in

1. the date he or she received the communication,
2. the communication itself, and
3. the security's past unique returns.

In the mathematical development that follows, information about t_M contained in the communication is ignored. Its incorporation into the analysis is straightforward and is discussed briefly later.

IV. The Probability Distribution of the Market Date

Letting X_t represent the security's unique return (in continuous form) for the period ended at time t, the probability of interest is

$$P(t_M \mid X_{-\infty}, \cdots, X_{t_I} t_I). \tag{1}$$

Using Bayes theorem[6], this can be written as

$$P(t_M \mid X_{-\infty}, \cdots, X_{t_I} t_I) = \frac{P(t_M \mid t_I) P(X_{-\infty}, \cdots X_{t_I} \mid t_M t_I)}{\sum_{t_M=-\infty}^{\infty} P(t_M \mid t_I) P(X_{-\infty}, \cdots, X_{t_I} \mid t_M t_I)}. \tag{2}$$

[6] Bayes theorem ordinarily is written as

$$P(A_k \mid B) = \frac{P(A_k) P(B \mid A_k)}{\sum_i P(A_i) P(B \mid A_i)}. \tag{a}$$

Associating A_i with t_M, B with a particular sequence of past unique returns $(X_{-\infty}, \cdots, X_{t_I})$, and C with a particular t_I, Expression (1) can be written as $P(A_k \mid BC)$ and Equation (2) can be written as

$$P(A_k \mid BC) = \frac{P(A_k \mid C) P(B \mid A_k C)}{\sum_i P(A_i \mid C) P(B \mid A_i C)}. \tag{b}$$

Equation (b) is exactly the same as (a) except for the additional conditioning variable C. To see that version (b) is valid, first note that a fundamental theorem of probability is

$$P(ABC) = P(A) P(BIA) P(CIAB). \tag{c}$$

In this paper, it is assumed that the unique return associated with all events other than the one under consideration has a normal distribution with mean zero and variance σ^2. This eliminates the need for the investor to have total knowledge of events. Thus,

$$P(X_{-\infty}, \cdots, X_{t_I} \mid t_M t_I) = \prod_{i=-\infty}^{t_I} \left[\frac{e^{-X_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma} \right], \quad t_M > t_I \tag{3}$$

and

$$P(X_{-\infty}, \cdots, X_{t_I} \mid t_M t_I) = \prod_{i=-\infty}^{t_I} \left[\frac{e^{-(X_i - \delta_i t_M V)^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma} \right], \quad t_M \leq t_I. \tag{4}$$

The expression on the right-hand side of (4) can be rewritten as

$$e^{-(X_{t_M} - V)^2/2\sigma^2} e^{X_{t_M}^2/2\sigma^2} \prod_{i=-\infty}^{t_I} \left[\frac{e^{-X_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma} \right].$$

But

$$-(X_{t_M} - V)^2 + X_{t_M}^2 = -X_{t_M}^2 + 2VX_{t_M} - V^2 + X_{t_M}^2,$$
$$= 2VX_{t_M} - V^2,$$

Since the order of these events is immaterial,

$$P(ABC) = P(C)P(B \mid C)P(A \mid BC)$$

and

$$P(ABC) = P(C)P(AIC)P(B \mid AC). \tag{d}$$

Setting the two right-hand sides of (d) equal and solving for $P(A \mid BC)$ gives

$$P(A \mid BC) = \frac{P(A \mid C)P(B \mid AC)}{P(B \mid C)}. \tag{e}$$

Rewriting A as A_k, (e) becomes

$$P(A_k \mid BC) = \frac{P(A_k \mid C)P(B \mid A_k C)}{P(B \mid C)}. \tag{f}$$

The evaluation of $P(B \mid C)$ proceeds as follows:

$$P(BC) = \textstyle\sum_i P(A_i BC),$$
$$P(C)P(B \mid C) = \textstyle\sum_i P(C)P(A_i \mid C)P(B \mid A_i C),$$
$$P(B \mid C) = \textstyle\sum_i P(A_i \mid C)P(B \mid A_i C). \tag{g}$$

Thus

$$P(A_k \mid BC) = \frac{P(A_k \mid C)P(B \mid A_k C)}{\sum_i P(A_i \mid C)P(B \mid A_i C)}$$

and Equation (2) is valid.

and (4) becomes

$$P(X_{-\infty}, \cdots, X_{t_I} \mid t_M t_I) = e^{-V^2/2\sigma^2} e^{VX_{t_M}/\sigma^2} \prod_{i=-\infty}^{t_I} \left[\frac{e^{-X_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\right], \quad t_M \leq t_I. \tag{5}$$

The calculation of $P(t_M \mid t_I)$ requires three steps. The first step consists of using Bayes theorem to obtain $P(t_E \mid t_I)$ starting with a prior on t_E, $P(t_E)$, and the structure in Figure 4. The second step consists of using the structure in Figure 4 to obtain $P(t_M \mid t_I t_E)$. The last step consists of combining the distributions obtained in the first two steps using

$$P(t_M \mid t_I) = \textstyle\sum_{t_E} P(t_E \mid t_I) P(t_M \mid t_I t_E). \tag{6}$$

In this paper, t_E is assumed to have the following distribution.

$$P(t_E) = \left(\frac{1}{\delta}\right) \quad t_E = \left(T_0 - \frac{\delta}{2} + 1\right), \cdots, \left(T_0 + \frac{\delta}{2}\right). \tag{7}$$

If the communication contains information about t_M, it will usually be in the form of specific information about t_E. This can be incorporated into the analysis by choosing an appropriate prior for t_E. Suppose, for example, that the communication consists of the statement that a particular event occurred during a specific week but does not include the exact date. Then by choosing T_0 and δ correctly, a uniform prior with positive weight for only the days of that week could be constructed. More generally, the prior should take whatever form is necessary to reflect adequately the nature of the information about t_M contained in the communication.

Using Bayes theorem,

$$P(t_E \mid t_I) = \frac{P(t_E) P(t_I \mid t_E)}{\sum_{t_E = T_0 - (\delta/2)+1}^{t_I - 1} P(t_E) P(t_I \mid t_E)}. \tag{8}$$

The structure in Figure 4 implies that

$$P(t_I \mid t_E) = \alpha(1-\alpha)^{t_I - 1 - t_E}, \quad t_I > t_E. \tag{9}$$

Substituting (7) and (9) in (8) results in

$$P(t_E \mid t_I) = \frac{(\alpha/\delta)(1-\alpha)^{t_I - 1 - t_E}}{\sum_{t_E = T_0 - (\delta/2)+1}^{t_I - 1} (\alpha/\delta)(1-\alpha)^{t_I - 1 - t_E}},$$

$$= \frac{\alpha(1-\alpha)^{t_I - 1 - t_E}}{\alpha \sum_{i=0}^{t_I - T_0 - 2 + (\delta/2)} (1-\alpha)^i},$$

$$P(t_E \mid t_I) = \frac{\alpha(1-\alpha)^{t_I - 1 - t_E}}{[1 - (1-\alpha)^{t_I - T_0 - 1 + (\delta/2)}]}. \tag{10}$$

By choosing T_0 properly, the investor can be sure that $(t_I - t_0 - 1)$ is bounded. In this case,

$$\lim_{\delta \to \infty} P(t_E \mid t_I) = \alpha(1-\alpha)^{t_I - 1 - t_E}, \quad t_E < t_I. \tag{11}$$

Because the structure of Figure 4 implies independence between t_M and t_I except through t_E,

$$P(t_M \mid t_I t_E) = P(t_M \mid t_E), \tag{12}$$

$$P(t_M \mid t_I t_E) = \gamma\,(1-\gamma)^{t_M-1-t_E}. \tag{13}$$

Substituting (11) and (13) in (6),

$$P(t_M \mid t_I) = \textstyle\sum_{t_E=-\infty}^{t_{MI}-1} \alpha(1-\alpha)^{t_I-1-t_E}\gamma(1-\gamma)^{t_M-1-t_E},$$

$$t_{MI} = \min(t_M, t_E). \tag{14}$$

When $t_M \le t_I$, (14) is

$$P(t_M \mid t_I) = \textstyle\sum_{t_E=-\infty}^{t_M-1} \gamma\alpha(1-\alpha)^{t_I-t_M}[(1-\gamma)(1-\alpha)]^{t_M-1-t_E},$$

$$= \gamma\alpha(1-\alpha)^{t_I-t_M} \textstyle\sum_{i=0}^{\infty} [(1-\gamma)(1-\alpha)]^i,$$

$$P(t_M \mid t_I) = \frac{\gamma\alpha(1-\alpha)^{t_I-t_M}}{[1-(1-\gamma)(1-\alpha)]}, \quad t_M \le t_I. \tag{15}$$

When $t_M > t_I$, (14) is

$$P(t_M \mid t_I) = \textstyle\sum_{t_E=-\infty}^{t_I-1} \gamma\alpha(1-\gamma)^{t_M-t_I}[(1-\gamma)(1-\alpha)]^{t_I-1-t_E},$$

$$= \gamma\alpha(1-\gamma)^{t_M-t_I} \textstyle\sum_{i=0}^{\infty} [(1-\gamma)(1-\alpha)]^i,$$

$$P(t_M \mid t_I) = \frac{\gamma\alpha(1-\gamma)^{t_M-t_I}}{[1-(1-\gamma)(1-\alpha)]}, \quad t_M > t_I. \tag{16}$$

Equation (2) can be evaluated using (4), (5), (15), and (16). The denominator of (2) is found as follows

$$\textstyle\sum_{t_M=-\infty}^{\infty} P(t_M \mid t_I)P(X_{-\infty}, \cdots, X_{t_I} \mid t_M t_I)$$

$$= \textstyle\sum_{t_M=-\infty}^{t_I} P(t_M \mid t_I)P(X_{-\infty}, \cdots, X_{t_I} \mid t_M t_I)$$

$$+ \textstyle\sum_{t_M=t_I+1}^{\infty} P(t_M \mid t_I)P(X_{-\infty}, \cdots, X_{t_I} \mid t_M t_I). \tag{17}$$

The first sum on the right-hand side of (17) is

$$\textstyle\sum_{t_M=-\infty}^{t_I} \frac{\gamma\alpha(1-\alpha)^{t_I-t_M}}{[1-(1-\gamma)(1-\alpha)]} e^{-V^2/2\sigma^2} e^{VX_{t_m}/\sigma^2} \left[\prod_{i=-\infty}^{t_I} \frac{e^{-X_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\right],$$

$$= \frac{\gamma\alpha e^{-V^2/2\sigma^2}}{[1-1-\gamma)(1-\alpha)]} \left[\prod_{i=-\infty}^{t_I} \frac{e^{-X_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\right] \textstyle\sum_{t_M=-\infty}^{t_I} (1-\alpha)^{t_I-t_M} e^{VX_{t_M}/\sigma^2}. \tag{18}$$

The second sum on the right-hand side of (17) is

$$\textstyle\sum_{t_M=t_I+1}^{\infty} \frac{\gamma\alpha(1-\gamma)^{t_M-t_I}}{[1-(1-\gamma)(1-\alpha)]} \left[\prod_{i=-\infty}^{t_I} \frac{e^{-X_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\right], \tag{19}$$

$$= \frac{\alpha(1-\gamma)}{[1-(1-\gamma)(1-\alpha)]} \left[\prod_{i=-\infty}^{t_I} \frac{e^{-X_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\right]. \tag{20}$$

For the case $t_M \leq t_I$, the numerator of (2) is a single term from the sum (18)

$$\frac{\gamma\alpha e^{-V^2/2\sigma^2}}{[1-(1-\gamma)(1-\alpha)]}\left[\prod_{i=-\infty}^{t_I}\frac{e^{-X_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\right](1-\alpha)^{t_I-t_M}e^{VX_{t_M}/\sigma^2}. \quad (21)$$

Thus,

$$P(t_M \mid X_{-\infty}, \cdots, X_{t_I}t_I) = \frac{\gamma e^{-V^2/2\sigma^2}(1-\alpha)^{t_I-t_M}e^{VX_{t_M}/\sigma^2}}{(1-\gamma)+\gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}}, \quad t_M \leq t_I. \quad (22)$$

For the case $t_M > t_I$, the numerator of (2) is a single term from the sum (19), and

$$P(t_M \mid X_{-\infty}, \cdots, X_{t_I}t_I) = \frac{\gamma(1-\gamma)^{t_M-t_I}}{(1-\gamma)+\gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}}, \quad t_M > t_I. \quad (23)$$

Two immediate consequences of (22) and (23) are

$$P(t_M \leq t_I \mid X_{-\infty}, \cdots, X_{t_I}t_I) = \frac{\gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}}{(1-\gamma)+\gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}e^{VX_j/\sigma^2}}; \quad (24)$$

and

$$P(t_M > t_I \mid X_{-\infty}, \cdots, X_{t_I}t_I) = \frac{(1-\gamma)}{(1-\gamma)+\gamma e^{-V^2/2\sigma^2}\sum_{j=-\infty}^{t_I}(1-\alpha)^{t_I-j}\,eVX_{j/\sigma^2}}. \quad (25)$$

Equation (25) provides the investor with the information needed to make an intelligent decision about establishing a position in the security.

To see how Equation (25) can be used, suppose that γ and α both are 0.5 per day. This means that for both the investor and the market, the probability is 0.5 that they will receive the information tomorrow if they did not receive it today. Also, assume that the value of the information is about 2.0% of the price of the stock, so that $V = 0.02$. Finally, suppose that the security's standard deviation of daily unique return is about 1.0%, so that $\sigma = 0.01$.

Table II contains a hypothetical record of past unique returns, X_j, for the security. Each return is a draw from a normal distribution with mean 0.0 and standard deviation 0.01. Thus, none of these past unique returns reflects the impact of the information, and the market date lies in the future.

An examination of Table II suggests that the market date could well have been in the past. Two days prior to the investor date, a daily unique return of almost 1.0% was experienced. This is suggestive of an unusual event. The same can be said of the unique returns experienced three and ten days prior to the investor date.

Table III contains an evaluation of Equation (25). Note how rapidly the contribution of the terms in the sum in the denominator declines. Although only 30 terms are included, it is clear that no significant accuracy has been lost. This

Table II

Hypothetical Unique Returns

$(t_I - j)$	X_j	$(t_I - j)$	X_j
0	−0.0107	15	−0.0146
1	0.0022	16	−0.0120
2	0.0092	17	0.0079
3	0.0083	18	0.0077
4	−0.0206	19	−0.0133
5	−0.0018	20	−0.0060
6	−0.0110	21	0.0006
7	0.0035	22	0.0049
8	−0.0106	23	−0.0105
9	−0.0070	24	−0.0063
10	0.0098	25	0.0024
11	0.0028	26	−0.0073
12	−0.0101	27	−0.0031
13	0.0229	28	−0.0057
14	−0.0043	29	−0.0090

behavior reflects the rapid decline of the powers of the fraction $(1 - \alpha)$. Thus, it is typical.

For this hypothetical, but realistic example, the probability that the market has not yet received the information is about 70%. This implies a probability of 30% that the market has received the information. Statistically, the case is remarkably clear cut. An intuitive analysis of Table II, on the other hand, is unrewarding. The usefulness of the technique is evident.

A more sophisticated investment strategy is presented in the next section. Two additional consequences of Equations (22) and (23) which will be needed there are

$$P(t_I < t_M \le t_I + H \mid X_{-\infty}, \cdots, X_{t_I} t_I)$$

$$= \sum_{t_M=t_I+1}^{t_I+H} \frac{\gamma(1-\gamma)^{t_M-t_I}}{(1-\gamma) + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}},$$

$$P(t_I < t_M \le t_I + H \mid X_{-\infty}, \cdots, X_{t_I} t_I)$$

$$= \frac{(1-\gamma)[1-(1-\gamma)^H]}{(1-\gamma) + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}}; \quad (26)$$

and

$$P(t_M > t_I + H \mid X_{-\infty}, \cdots, X_{t_I} t_I)$$

$$= \sum_{t_M=t_I+H+1}^{\infty} \frac{\gamma(1-\gamma)^{t_M-t_I}}{(1-\gamma) + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}},$$

$$P(t_M > t_I + H \mid X \cdots, X_{t_I} t_I)$$

$$= \frac{(1-\gamma)^{H+1}}{(1-\gamma) + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}}. \quad (27)$$

Table III

Computation of the Probability that the Market Date is later than the Investor Date

$(t_I - j)$	$(1-\alpha)^{t_I - j}$	$\frac{VX_j}{\sigma^2}$	e^{VX_j/σ^2}	$(1-\alpha)^{t_I - j} e^{VX_j/\sigma^2}$	Cumulative Sum
0	1.0000000000	−2.140	0.1177	0.1176548430	0.1176548430
1	0.5000000000	0.440	1.5527	0.7763536093	0.8940084523
2	0.2500000000	1.840	6.2965	1.5741345650	2.4681430180
3	0.1250000000	1.660	5.2593	0.6574138556	3.1255568730
4	0.0625000000	−4.120	0.1624	0.0010152822	3.1265721550
5	0.0312500000	−0.360	0.6977	0.0218023852	3.1483745400
6	0.0156250000	−2.200	0.1108	0.0017312993	3.1501058400
7	0.0078125000	0.700	2.0138	0.0157324430	3.1658382830
8	0.0039062500	−2.120	0.1200	0.0004688735	3.1663071560
9	0.0019531250	−1.400	0.2466	0.0004816347	3.1667887910
10	0.0009765625	1.960	7.0993	0.0069329366	3.1737217280
11	0.0004882813	0.560	1.7507	0.0008548206	3.1745765480
12	0.0002441406	−2.020	0.1327	0.0000323866	3.1746080350
13	0.0001220703	4.580	97.5144	0.0119036126	3.1865125470
14	0.0000610352	−0.860	0.4232	0.0000258278	3.1865383750
15	0.0000305176	−2.920	0.0539	0.0000016459	3.1865400210
16	0.0000152588	−2.400	0.0907	0.0000013842	3.1865414050
17	0.0000076294	1.580	4.8550	0.0000370404	3.1865784460
18	0.0000038147	1.540	4.6646	0.0000177940	3.1865962400
19	0.0000019073	−2.660	0.0699	0.0000001334	3.1865963730
20	0.0000009537	−1.200	0.3012	0.0000002872	3.1865966600
21	0.0000004768	0.120	1.1275	0.0000005376	3.1865971980
22	0.0000002384	0.980	2.6645	0.0000006353	3.1865978330
23	0.0000001192	−2.100	0.1225	0.0000000146	3.1865978480
24	0.0000000596	−1.260	0.2837	0.0000000169	3.1865978650
25	0.0000000298	0.480	1.6161	0.0000000482	3.1865979130
26	0.0000000149	−1.460	0.2322	0.0000000035	3.1865979160
27	0.0000000075	−0.620	0.5379	0.0000000040	3.1865979200
28	0.0000000037	−1.140	0.3198	0.0000000012	3.1865979220
29	0.0000000019	−1.800	0.1653	0.0000000003	3.1865979220

$$P(t_M > t_I/X_{-\infty}, \cdots, X_{t_I} t_I) = \frac{(1-0.5)}{(1-0.5) + (0.5)(0.1353352832)(3.1865979220)}$$

$$P(t_M > t_I/X_{-\infty}, \cdots, X_{t_I} t_I) = 0.699$$

V. Capitalizing on the Probability Distribution of the Market Date

In selecting a portfolio strategy which permits the investor to capitalize on the information received, it is important to remember that the impact of the information on the price of the security is in the nature of a unique return in the Treynor-Black sense. If the investor uses the Treynor-Black approach to portfolio construction, he or she will be interested in that portion of the probability distribution of unique return associated with the communication. In what follows, it is assumed that the investor either purchases or sells short the security at time t_I and liquidates the position H time units later at time $t_I + H$. The investor is assumed to be a Treynor-Black investor whose investment horizon is H time units in the future. Denoting that portion of the unique return associated with the communication by r,

$$P(r = 0 \mid X_{-\infty}, \cdots, X_{t_I}t_I) = P(t_M \le t_I \mid X_{-\infty}, \cdots, X_{t_I}t_I) + P(t_M > t_I + H \mid X_{-\infty}, \cdots, X_{t_I}t_I) \quad (28)$$

and

$$P\left(r = \frac{V}{H} \,\middle|\, X_{-\infty}, \cdots, X_{t_I}t_I\right) = P(t_I < t_M \le t_I + H \mid X_{-\infty}, \cdots, X_{t_I}t_I). \quad (29)$$

The mean of r is

$$\mu_r = P\left(r = \frac{V}{H} \,\middle|\, X_{-\infty}, \cdots, X_{t_I}t_I\right)\left(\frac{V}{H}\right). \quad (30)$$

The variance of r is

$$\begin{aligned}
\sigma_r^2 &= \varepsilon(r^2) - \mu_r^2, \\
&= P\left(r = \frac{V}{H} \,\middle|\, X_{-\infty}, \cdots, X_{t_I}t_I\right)\left(\frac{V}{H}\right)^2 \\
&\quad - \left[P\left(r = \frac{V}{H} \,\middle|\, X_{-\infty}, \cdots, X_{t_I}t_I\right)\right]^2\left(\frac{V}{H}\right)^2, \\
&= \left(\frac{V}{H}\right)^2 P\left(r = \frac{V}{H} \,\middle|\, X_{-\infty}, \cdots, X_{t_I}t_I\right) \\
&\quad \cdot \left[1 - P\left(r = \frac{V}{H} \,\middle|\, X_{-\infty}, \cdots, X_{t_I}t_I\right)\right], \\
\sigma_r^2 &= \left(\frac{V}{H}\right)^2 P\left(r = \frac{V}{H} \,\middle|\, X_{-\infty}, \cdots, X_{t_I}t_I\right) \\
&\quad \cdot P(r = 0 \mid X_{-\infty}, \cdots, X_{t_I}t_I). \qquad (31)
\end{aligned}$$

Equations (28), (29), (30), and (31) are easily evaluated using equations (24), (26), and (27). In Treynor-Black terminology, (30) is the appraisal premium associated with the communication and (31) is its associated appraisal variance. This portion of the unique return over the interval from t_I to $(t_I + H)$ is proportional to a point binomial random variable.

Denoting the remaining portion of the unique return over the interval from t_I to $(t_I + H)$ by R,

$$R = \frac{1}{H} \textstyle\sum_{j=t_I+1}^{t_I+H} Y_j. \quad (32)$$

By assumption,

$$P(Y_{t_I+1}, \cdots, Y_{t_I+H}) = \prod_{i=t_I+1}^{t_I+H} \left[\frac{e^{-Y_i^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\right]. \quad (33)$$

Since R is a weighted sum of independent normal variables, it also has a normal distribution with mean and variance given by

$$\mu_R = 0.0$$

and

$$\sigma_R^2 = \frac{\sigma^2}{H}. \tag{34}$$

Thus,

$$P(R) = \frac{e^{(-R^2/2\sigma^2/H)}}{\sqrt{2\pi}\,\sigma/\sqrt{H}}. \tag{35}$$

The total unique return, ρ, over the interval from t_I to $(t_I + H)$ is simply

$$\rho = R + r \tag{36}$$

where R and r are independent. The joint probability distribution of R and r is

$$P(R, r \mid X_{-\infty}, \cdots, X_{t_I}t_I) = P(R)P(r \mid X_{-\infty}, \cdots, X_{t_I}t_I). \tag{37}$$

Since r is proportional to a point binomial and R is normal, the probability distribution for ρ is

$$\begin{aligned} P(\rho \mid X_{-\infty}, \cdots, X_{t_I}t_I) &= P(r = 0 \mid X_{-\infty}, \cdots, X_{t_I}t_I)P(R = \rho) \\ &+ P\left(r = \frac{V}{H} \,\middle|\, X_{-\infty}, \cdots, X_{t_I}t_I\right)P\left(R = \rho - \frac{V}{H}\right). \end{aligned} \tag{38}$$

Equation (38) shows that ρ has a bimodal distribution with one mode at 0.0 and the other at (V/H). This distribution is the weighted sum of two normal distributions with identical variances (σ^2/H) and means of 0.0 and (V/H). Furthermore, since all the variables involved are independent and all returns are expressed in continuous form,

$$\mu_\rho = \mu_R + \mu_r = \mu_r \tag{39}$$

and

$$\sigma_\rho^2 = \sigma_R^2 + \sigma_r^2. \tag{40}$$

The investor's appraisal premium is given by (39), the appraisal variance by (40), and the appraisal ratio, γ_ρ^2, by

$$\gamma_\rho^2 = \frac{\mu_\rho^2}{\sigma_\rho^2}, \tag{41}$$

$$\gamma_\rho^2 = \frac{\mu_r^2}{\sigma_R^2 + \sigma_r^2}. \tag{42}$$

Making all the appropriate substitutions,

$$\mu_\rho = \mu_r = \frac{\left(\frac{V}{H}\right)(-\alpha)[1 - (1 - \gamma)^H]}{(1 - \gamma) + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1 - \alpha)^{t_I - j} e^{VX_j/\sigma^2}}, \tag{43}$$

$$\sigma_\rho^2 = \sigma^2/H + (V/H)^2 \cdot \frac{(1-\gamma)[1-(1-\gamma)^H] \, [(1-\gamma)^{H+1} + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}]}{[(1-\gamma) + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}]^2}, \quad (44)$$

$$P(\rho \mid X_{-\infty}, \cdots, X_{t_I} t_I)$$

$$= \left[\frac{(1-\gamma)^{H+1} + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}}{(1-\gamma) + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}}\right] \frac{e^{-\rho^2/2(\sigma^2/H)}}{\sqrt{2\pi}\,(\sigma/\sqrt{H})}$$

$$+ \left[\frac{(1-\gamma)[1-(1-\gamma)^H]}{(1-\gamma) + \gamma e^{-V^2/2\sigma^2} \sum_{j=-\infty}^{t_I} (1-\alpha)^{t_I-j} e^{VX_j/\sigma^2}}\right] \frac{e^{-(\rho - V/H)^2/2(\sigma^2/H)}}{\sqrt{2\pi}(\sigma/\sqrt{H})} \quad (45)$$

VI. Conclusions

Adherents of technical analysis claim that unusual profit can be achieved using only past security prices. Most academics believe that the securities markets are efficient enough to make this impossible. This paper has shown that past prices, when combined with other valuable information, can indeed be helpful in achieving unusual profit. However, it is the nonprice information that creates the opportunity. The past prices serve only to permit its efficient exploitation.

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DISCUSSION

ERIC H. SORENSEN*: This paper by Jack Treynor and Robert Ferguson (TF) is an attempt to demonstrate the usefulness of knowing past price information in making investment decisions. The normative contribution of the paper is the development of a Bayesian probability estimate to assess using past price data whether or not the market has already incorporated some firm-specific information, which has been made available to the investor. The obvious implication is that if the market *has not* discovered the information, and this can be confirmed with past price data, then the holder of private information can act accordingly.

TF conclude: "This paper has shown that past prices, when combined with other valuable information, can indeed be helpful in achieving unusual profit. However, it is the nonprice information that creates the opportunity. The past prices serve only to permit its efficient exploitation."

I have considerable pragmatic compassion for the notion. I have worn my money manager hat enough to come to believe that the examination of past *and*

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