Monthly Moving Averages--An Effective Investment Tool?

F. E. James, Jr.

MONTHLY MOVING AVERAGES--AN EFFECTIVE INVESTMENT TOOL?

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Analysts and investment advisors have long searched for investment tools that would either furnish predictive probabilities for future security price movements, or would aid in minimizing losses. One such tool, often recommended by market practitioners, is the Moving Average. This article describes a series of experiments that were performed upon actual market data, using Moving Averages of different lengths and weights, and presents results of the experiments. Conclusions derived from these experiments are suggested.

The investigation was carried out in conjunction with a larger number of experiments, designed to test the random walk hypothesis in security prices. The hypothesis holds that all price changes are serially independent; that "trends" are spurious or imaginary manifestations; and that tools of technical analysis, such as charts and the Dow Theory, are without investment value. Earlier findings by mathematicians, such as Kendall and Osborne, have supported the hypothesis.

If changes in prices are completely independent of all previous changes, there is no decision rule (based solely on previous changes) which may allow one to consistently "forecast" the future. ¹ Assuming the random walk

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**The views expressed herein are those of the author and do not necessarily reflect the views of the U. S. Air Force or the Department of Defense.

¹The expressions "predict the future" and "forecast" are used in this paper because they have come to be accepted terminology in the random walk literature. However, they are actually somewhat misleading; after long observation, a bystander might be able to accurately "forecast" the probability of the event: "black on the next ball" occurring on a roulette wheel. This would be true even though successive spins of the wheel are independent.

What is required as proof of dependence, for either a roulette wheel and a change in security price, is to demonstrate that the conditional probability of a future event, given information about past events, is not the same as the unconditional probability of the same future event, given no previous knowledge.
hypothesis to be true, no decision rule may be much more profitable than a simple "buy and hold" rule, where sales are made in one time period and repurchases in succeeding time periods (if dividends are added in, as they are here, and if the economy enjoys the type of secular uptrend observed during the past forty years).  

I. The Data

The basic data file for this study is a tape of month-end common stock price relatives, covering the period 1926–1960. These data were developed by the Center for Research in Security Prices at the University of Chicago. The data-tape was purchased by the School of Management, Rensselaer Polytechnic Institute, for use in this and related studies.  

The Center for Research in Security Prices was created in 1960 with a grant from Merrill, Lynch, Pierce, Fenner and Smith. Its compilation of monthly security prices is considered by some authorities to be one of the most comprehensive and accurate financial data-gathering projects ever executed. Complete details of the data collection and evaluation programs are included in a recent issue of The Journal of Business. However, because some of the items in this article are of special interest to this study, selected extracts are included below:

The common stocks included in the study are those listed and traded on the New York Stock Exchange during 1926–1960. Our data cover stocks only during the period of their listing. Several issues which were seldom or never traded on the Exchange during the period have effectively been excluded from the study.  

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2This fact is not strictly true; the investor who sells at price $X_0$ and repurchases the same security later at price $X_0 + e$ or $X_0 - e$, with equal probability of either event, will improve his position. However, the "loss" of dividends and the drift of prices upward offset this tendency (reduce the probability of repurchasing at $X_0 - e$) to the extent that it is doubtful that the investor will do better than buying and holding.

3I should like to express my appreciation to Dean Karger and to the faculty of the Rensselaer School of Management for their outstanding support throughout this project.

The prices consist of the last price on the last business day of the month, whenever such prices are available. When there was no trading of a particular stock on the last business day of the month, we use the mean of the bid-and-asked price or the bid price or the asked price.\textsuperscript{5}

We suspect errors in approximately 0.5 percent of the prices in our file. A substantial proportion of these errors are believed to be caused by errors in our sources. All errors are believed to be small and unbiased.\textsuperscript{6}

Two types of prices may be reconstructed from the data-tape. One type adjusts for dividends by adding in the value of the cash or property dividend received per share back to the price at the end of the month before computing the price relative for the month. This type of price, adjusted for dividends, will be labeled PRL data. The other price relative series ignores cash dividends, taxable subscription rights, and taxable distributors of other securities. This type of data, not adjusted for dividends, will be labeled PR2 data.

\section*{II. Theoretical Considerations}

An integral part of the technical hypothesis of stock price movements is that trends tend to persist; simply stated, this belief holds that when a security price rises or falls "in a decisive manner," the probabilities are greater than even that the rise or fall will continue for some time. The definition of "decisive" is at present a matter of some controversy among market professionals.

Among the most explicit of the technicians are those who define trends in terms of moving averages. For example, Tomlinson reports:

The one general category of "technical" methods which may have special interest for formula investors is the trend-detecting approach to market timing, which consists of determining, in one way or another, whether the trend of stock prices is up or down and following that trend until an opposite indication is given.\textsuperscript{7}

\textsuperscript{5}Ibid.

\textsuperscript{6}Ibid.

Moving averages can be used in various ways. A simple method is to wait until the actual market index crosses through the moving average of the same index, either up or down. If up, that is an indication the trend is up and stocks should be purchased; if down, the trend is down and stocks should be sold.8

Like the Dow Theory, moving average methods tell nothing about how far a market trend will continue in the direction indicated. For that reason, every signal must be obeyed. With this particular moving average method, there would have been a series of losses on the 1943–1944 period ... Every moving average method has to strike a compromise. The longer the period of time used for the average, the smoother is the resulting trend line and the less frequent are the false signals, or whipsawing transactions. But at the same time, the further from the actual turning points are the "buy" and "sell" indications given.9

Merritt supports Tomlinson's observations:

There are other methods of determining the major trend, one of them being to construct a moving average of stock prices. A moving average is simply a smooth curve which irons out the steep hills and valleys in the price charts. A definite change in trend is required before it appears on such a curve. One such system is based on a 12-month moving average of the Dow-Jones Industrials, plotted monthly. The trend of the market is assumed to be up when the Dow-Jones Industrials are above the 12-month moving average and the line is moving upward.10

Many authors have recommended that moving averages be considered in investment decision-making.11,12,13 Tomlinson notes that an open end

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investment fund has been founded, whose chief technique is said to be the Moving Average. One of the newer investment advisory services features Moving Averages as a primary aid. It is clear that Moving Averages deserve the investment analyst's serious consideration.

The simplest type of Moving Average is the unweighted average of a number of past closing prices. The formula for one such simple Moving Average is \[ \bar{x}_t = \frac{1}{N} \sum_{t-N}^{t} x_t \] where \( x_t \) is the price at time \( t \), and \( N \) the number of observations (prices). Some authorities recommend that this computed Moving Average be advanced one to three time periods; that is, that a buy or sell decision be made for the present period on the basis of the Moving Average value computed for the period ending one to three periods prior. A very general recommendation for use of Moving Averages is afforded by the following quote of Barnes: "When a stock's price curve penetrates its moving average line on the downside, this is a sell signal; when it penetrates its moving average on the upside, that is a buy signal." \(^{14}\)

Often the Moving Average recommendation is coupled with a tactic designed to delay action and to reduce the number of transactions (especially "whiplash" transactions) which may occur if the security fluctuates within a narrow price range for any appreciable time period. Thus the above rule might be modified to read: When a stock's price curve penetrates its moving average line on the downside by at least 5 percent, this is a sell signal; when it penetrates its moving average line on the upside by at least 5 percent, this is a buy signal. (Different percent penetrations have been recommended by the various authorities; 5 percent may be most common, but 2 percent and 3 percent values have been mentioned.)

The rationale behind the Moving Average is simple. The average is nothing more than a centering of past prices. \(^{15}\) If a security's present

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\(^{15}\) It is instructive to note that the simple moving average is nothing more than an intercept fitted by the method of least square to successive time periods. See Taro Yamane, *Statistics, An Introductory Analysis* (New York, N. Y.: Harper and Row, 1967), p. 860.
price moved above that level it achieved in the past, this is taken as an indication that an "uptrend" has been initiated. The investor seeks to take advantage of this trend by initiating purchase at an early date. Likewise, when the current price of a security moves below its Moving Average, this is taken as an indication that a "downtrend" is initiated, and appropriate investment action is recommended.

It is a fact, well known to statisticians, that the variance of an average of sample observations decreases as the number of observations increase. In a somewhat similar fashion, it is possible to observe that the Moving Average's response to new observations is somewhat slower where the Moving Average is composed of a large number of observations. One way of adjusting the volatility of the Moving Average is to decrease or increase the number of prices considered. One would, of course, expect that the variance of a 3-month Moving Average would be greater than that of a 10-month Moving Average. For this research project, it was determined that a 7-month Moving Average corresponded most closely to the "200-day Moving Average" that has been recommended by many authorities, and all unweighted Moving Averages utilized were constructed for an observational period of 7 months.\(^{16}\)

Another type of Moving Average utilized in this research is the Exponentially Smoothed Moving Average. The general principle underlying this average is much the same as outlined in the preceding discussion, but there are a few important differences. Exponentially Smoothed averages do not require that a complete file of previous prices be maintained, as in the case of the unweighted moving averages. The Exponential formula permits one to approximate an unweighted moving average of any length by a simple adjustment of one of the variables in the formula, the \(\alpha\) factor. Table 1 below lists smoothing constants so that the average age of data in the exponential model corresponds to the same average age in an N period unweighted moving average:\(^{17}\)

\(^{16}\) Some authorities who use or present information relative to 200-day Moving Averages are Granville, \textit{op. cit.}, p. 237, and Barnes, \textit{op. cit.}, p. 182.

\(^{17}\) Barnes, \textit{op. cit.}, p. 108. Note that although the average age of the two methods is identical, they may yield somewhat different answers.
Table 1

<table>
<thead>
<tr>
<th>Number of Observations in an Unweighted Moving Average</th>
<th>Equivalent Smoothing Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>α</td>
</tr>
<tr>
<td>4</td>
<td>.4</td>
</tr>
<tr>
<td>5.67</td>
<td>.3</td>
</tr>
<tr>
<td>9</td>
<td>.2</td>
</tr>
<tr>
<td>19</td>
<td>.1</td>
</tr>
</tbody>
</table>

Perhaps the chief advantage of exponential smoothing is that this technique gives a greater weight to the more recent observations in a time series. Intuitively, one would believe that more recent observations should have more value, and conceptual advantages may accrue if one uses this model to investigate trend persistency.

The formula for a simple exponential smoothed moving average at time \( i \) is

\[
F_i = F_{i-1} + \alpha(X_i - F_{i-1}), \text{ where}
\]

\( F_i \) = The value associated with the moving average, at period \( i \).

\( \alpha \) = The smoothing constant.

\( X_i \) = The closing prices of the security at period \( i \).

III. Experimental Procedure

Complete descriptions of the models and experimental procedure are afforded in the work referenced earlier. However, it may be useful to quickly summarize some of the main elements in the model construction:

It was noted earlier that two different types of data were available for this investigation, one including automatic and costless dividend reinvestment, the other ignoring dividends. The models could be divided into two general classifications, one of which used unweighted moving average, and the other used the exponential smoothed moving averages of variable \( \alpha \) factors. Every model contains two variables, named MAX and MIN, which

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18 F. E. James, Jr., The Implications of Trend Persistency in Portfolio Management, a thesis submitted to the Faculty of the School of Management at the Rensselaer Polytechnic Institute, Troy, New York in June 1967.
correspond to the two threshold constraints surrounding the Moving Average line. For example, the Unweighted Moving Average models (Table 2, Lines 1 and 8) all incorporated MAX values of 1.02 and MIN values of .98. Thus we may note that, in these models, the monthly closing price for each month after the 7th month was compared to the Moving Average value. If the closing price ever rose above the Moving Average value by 2 percent or more (if it was 102 percent of the Moving Average value), this was taken as a buy signal, and if this security was not held in a "long" condition at the time, a simulated purchase was made at the current price. On the other hand, if the closing price ever fell below the Moving Average by 2 percent or more (if it was 98 percent of the Moving Average value), this was taken as a sell signal, and if the security was not held in a "short" position at the time, a simulated sale was made.

Securities sampled were generally those that had been listed for the longest time period; where the sample number was 233 or less (Lines 1, 5, 6, and 8), the securities had all been listed over the entire 35-year period. Where the sample number was 798 (Lines 2 and 7), the securities had been listed for 200 months or longer; a sample number of 1,376 corresponds to a listing of 49 months or longer.

For all models, each security was followed as though it were observed by an investor who could either take a position in the security, or hold cash; he was not allowed to invest funds in any other security. In every case, an initial period of time was used to set initial values of Moving Averages (six months were used for exponentially smoothed averages, seven months for unweighted averages). Then the computer was programmed so that a portfolio manager was "given" the sum of $100.00. For every succeeding month, an updated moving average was constructed. The value of the moving average was compared to the month-end price of the security. Where the price first moved up a sufficient amount to initiate a position (or down, where short positions were allowed), the program simulated a "buy" (or "short sale") of as many shares as could be purchased at the month-end close price. This position was held until the price movement reversed itself by an amount sufficient to trigger a reverse indicator, at which time the position was closed out, and the funds thus generated were utilized to initiate a position in the opposite direction (for example, from "long" to "short").
At the end of the last time period for which data were available, two
values were calculated: XPOP was the present worth of the portfolio managed
according to the moving average decision rules outlined above. YPOP was the
value the portfolio would have achieved if the manager had initiated a "buy
and hold" procedure at that month at which the first signal was given to
initiate a position under the Moving Average rule, regardless of whether it
was a "buy" or a "sell short" signal. Thus D, defined as the difference
between XPOP and YPOP (the managed versus the unmanaged portfolio), affords
a measure of the comparative efficiency of the Moving Average decision rule.

The "paired t test" was used to obtain a measure of statistical sig-
nificance of the results. For each of the 1, 2, ..., n securities sampled
in each experiment, a value of

$$D_i = XPOP_i - YPOP_i \quad (i = 1, 2, ..., n)$$

was computed. The sample average, was used as an estimate of \( \bar{D} \), the true (but unknown)
difference. The null hypothesis,

$$H_0: E(D) \leq 0 \quad (\text{the moving average decision rule is ineffective})$$

was tested against the alternate hypothesis,

$$H_1: E(D) > 0 \quad (\text{the moving average decision rule is effective and}
\text{ will produce superior results to a "buy and hold" procedure.})$$
The standardized value of \( \bar{D} \) approaches the Normal distribution; that is, the value of

$$\frac{\bar{D} - E(\bar{D})}{\sigma_{\bar{D}}} \sim N(0,1), \text{ as } n \to \infty. \quad 19$$

19 Where N(0,1) represents the standardized Normal variate. The size
of the sample required to make the approximation adequate for tests of sig-
ificance depends on the nature of the underlying population. As Miller
and Freund state: "In practice, the normal distribution provides an excel-
 lent approximation to the sampling distribution of \( x \) for \( n \) as small as 25
or 30, with hardly any restrictions on the shape of the population." I. Miller and J. Freund, Probability and Statistics for Engineers (Engle-
examples \( n \) is never less than 200.
Where \( \sigma^2 \) (the population variance) is unknown, it is possible to substitute \( s^2/n \) (the sample variance) and to form a statistic which follows the "t" distribution:

\[
t = \frac{\overline{D} - E(\overline{D})}{s/\sqrt{n}}
\]

When the sample number is very large, the convergence of the "t" distribution to the Normal is very close, and a test using Normal tables is acceptable.

IV. Results

Results of the experiments, along with descriptive parameters of the models, values of the sample standard error, \( \overline{D} \), and the t statistic are presented below.

The parts of this table that will be most interesting to the financial analyst are the columns marked "n" and "\( \overline{D} \)". The "\( \overline{D} \)" column shows the average difference between a portfolio managed according to the Moving Average decision rule, and one managed according to a "buy and hold" decision rule. The "n" column shows the number of securities examined. Thus, for instance, Line 2 shows that a sample of 798 securities were examined, and that a manager applying the moving average decision rule to these securities would have, on the average, been worse off by \$1,059.00 than if he had simply bought the security and held it. (Recall that the timing of the initial purchase was set at the first time a transaction was executed in the Moving Average portfolio, so that comparisons might be more accurately drawn.)

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20 The substitution is not direct, of course. The random variable formed by the ratio of a normalized random variable over the quantity obtained by taking the square root of an independent chi-square random variable divided by its degrees of freedom follows the "t" distribution with the degrees of freedom associated with the chi-square variate. Some algebraic manipulation of this quantity will yield a statistic in the form presented above. See Albert Bowker and Gerald Lieberman, *Engineering Statistics* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1964), p. 79.
Table 2

Using Data Adjusted for Automatic Dividend Reinvestment

<table>
<thead>
<tr>
<th>Name</th>
<th>α</th>
<th>MAX</th>
<th>MIN</th>
<th>n</th>
<th>s/√n</th>
<th>d</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. UNWEIGHTED MOVING AVERAGE (7 months)</td>
<td>1.02</td>
<td>.98</td>
<td>233</td>
<td>425</td>
<td>-771</td>
<td>-1.815</td>
<td></td>
</tr>
<tr>
<td>2. EXPONENTIAL SMOOTHING, LONG AND SHORT</td>
<td>.1</td>
<td>1.05</td>
<td>.95</td>
<td>798</td>
<td>184</td>
<td>-1059</td>
<td>-5.728</td>
</tr>
<tr>
<td>3. EXPONENTIAL SMOOTHING, LONG ONLY</td>
<td>.1</td>
<td>1.05</td>
<td>.95</td>
<td>1376</td>
<td>70</td>
<td>-409</td>
<td>-5.811</td>
</tr>
<tr>
<td>4. EXPONENTIAL SMOOTHING, LONG ONLY</td>
<td>.3</td>
<td>1.05</td>
<td>.95</td>
<td>1376</td>
<td>84</td>
<td>-422</td>
<td>-4.988</td>
</tr>
<tr>
<td>5. EXPONENTIAL SMOOTHING, LONG ONLY</td>
<td>.4</td>
<td>1.05</td>
<td>.95</td>
<td>232</td>
<td>470</td>
<td>-145</td>
<td>-0.309</td>
</tr>
</tbody>
</table>

Using Data Not Adjusted for Cash Dividends

<table>
<thead>
<tr>
<th>Name</th>
<th>α</th>
<th>MAX</th>
<th>MIN</th>
<th>n</th>
<th>s/√n</th>
<th>d</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. EXPONENTIAL SMOOTHING, LONG ONLY, PR2</td>
<td>.4</td>
<td>1.05</td>
<td>.95</td>
<td>232</td>
<td>188</td>
<td>337</td>
<td>1.788</td>
</tr>
<tr>
<td>7. EXPONENTIAL SMOOTHING, LONG AND SHORT, PR2</td>
<td>.1</td>
<td>1.05</td>
<td>.95</td>
<td>798</td>
<td>101</td>
<td>92</td>
<td>0.906</td>
</tr>
<tr>
<td>8. UNWEIGHTED MOVING AVERAGE, PR2</td>
<td>1.02</td>
<td>.98</td>
<td>233</td>
<td>154</td>
<td>136</td>
<td>0.887</td>
<td></td>
</tr>
</tbody>
</table>

It is easy to see that few of the decision rules would have beat a simple "buy and hold" philosophy. The largest dollar amount occurred in Line 6, but even here the average dollar difference was only $337. It is interesting to note only two of the differences were great enough (in comparison to their standard error) to be statistically significant, even at the 5 percent level, and none were significant at the 1 percent level.

V. Conclusions

One might conjecture at the reason why the results of the decision rule were so consistently inferior where the operational data were adjusted to include the effect of dividends. It appears that the forecasting ability of the decision rule may have been severely taxed with this type
of experiment, with a two-fold handicap: (1) The portfolio did not re-
ceive dividend income for those periods over which the rule called for no
securities to be held; (2) During these periods, the portfolio did not
participate in the secular uptrend in the economy and in the securities
market that has been observed over the past 40 years. The most inferior
results of the decision rule occurred where the model allowed both long and
short sales on the operational data. In this case, the managed portfolio
operated under the additional handicap of being required, in effect, to
"pay" for the value of dividends disbursed while a short position was
maintained.

Proponents of the monthly moving average technique might point out
that the "buy and hold" results may appear better than actually warranted,
because the period ended in an era of generally high stock prices (at least
by historic standards). This fact is, of course, quite true; yet the moving
average decision rules were operational over considerable periods of time,
embracing both high and low markets. The technique has been said to possess
enough predictive power to enable a user to sell somewhere near the price
peak (after it has passed) and repurchase near the bottom of the ensuing
decline. From the results of this rather extensive test, dealing with a
large sample and over considerable time periods, it cannot be said that the
monthly moving average technique has succeeded in its purpose; at least,
not where a real-world condition (stocks pay dividends) was observed.

It should be noted that this examination has dealt with monthly moving
averages only. It is entirely possible that moving averages of different
time periods—say daily, or weekly—would yield different results. The study
has not compared the variance of the two different portfolios; such an
evaluation might indeed reveal that the variance of the return of the moving
average method is smaller, thus indicating a possible decrease of risk when
using the technique. What does seem abundantly clear, however, is that when
records of individual stocks (as opposed to averages or indices of stock
prices) are examined, this survey detected little reason to believe that the
investor's position will be benefited by use of the monthly Moving Averages.