THE EMPIRICAL RELATIONSHIP BETWEEN TRADING VOLUME, RETURNS AND VOLATILITY

Timothy J. Brailsford

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Non-technical Executive Summary

There is substantial interest in how trading volume is related to price movements in the stock market. Clearly, positive trading volume is needed to generate observed market prices. A naive view of the market is that the greater the level of volume, the greater the price movement. However, instances can be found where a low level of volume is associated with large price movements and conversely, a high level of volume is associated with no change in price. Market folklore claims that the relationship between volume and price movements depends on whether the market is in a bull or bear run. In a bull market, a relatively higher level of volume is associated with a given price change in comparison to a bear market. However, these claims are anecdotal and unsubstantiated. This research paper examines the relationship between the level of trading volume and the magnitude of price changes, thereby providing scientific evidence on this topic in the Australian stock market.

The paper commences with a discussion of the general issues and a review of prior studies conducted on this topic, primarily in the USA. A formal model from this literature is then redeveloped which provides some testable implications. The main point about the model is that it predicts an asymmetric relationship between the level of trading volume and price change which is dependent on the direction of the change in price. Positive changes in price are hypothesised to be associated with a greater level of trading volume than negative price changes. That is, the theoretical model provides a prediction which is consistent with market folklore. The advantage of the model is that it provides an conceptually defensible explanation as to why an asymmetric relationship should exist.

A second model is also discussed which provides an indirect test for the link between price changes and information flow. The Central Limit Theorem (or law of large numbers) states that the distribution of many observations, including share price changes, will conform to the normal distribution. However, there is substantial prior evidence that share price changes do not conform to such a distribution. There are too many observations which fall into the extreme ends of the share price change distribution. That is, there are too many large positive price changes and too many large negative price changes. It is hypothesised that the non-normal distribution is due to the process of information arrival. If current information is related to past information, then price changes will also be related to past price changes. Hence, a large price change will tend to be followed by another large price change. The problem with this hypothesis is that the flow of information is very difficult to identify and quantify as a variable. However, if it is assumed that trading volume is induced by information flow, then volume can be used as a proxy variable.

To test the above hypotheses, daily data are collected for the aggregate market which is proxied by the All Ordinaries Index and for the top five ranked stocks over a minimum five-year period. An initial analysis indicates that trading volume is lowest on Mondays and grows during the week to Friday, which is the day with the heaviest volume. Regression analysis indicates that trading volume is positively related to price changes and that this relationship depends on the direction of the price change. Negative price changes are shown to be more sensitive to trading volume than positive price movements. Hence, there is support for an asymmetric relationship. This
relationship is strongest for the more volatile stocks. The results are robust to various model specifications and variable definitions.

The last section of the paper tests the information flow hypothesis using a econometric specification of volatility. The results show that volatility is positively related to trading volume and that much of the frequency in the extremes of the distribution of price changes can be accounted for by the level of trading volume.

The research paper should be of further interest to those with an interest in trading in the stock market.
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Abstract:

This paper presents an empirical analysis of the relationship between trading volume and stock return volatility in the Australian market. The initial analysis centres upon Karpoff's [1986, 1987] model of the volume-price change relationship. Evidence is found which supports the model. The relationship between price change and trading volume, irrespective of the direction of the price change, is significant across three alternative measures of daily trading volume for the aggregate market and individual stocks. Furthermore, evidence is found supporting the hypothesis that the volume-price change slope for negative returns is smaller than the slope for positive returns, thereby supporting an asymmetric relationship. Trading volume is then examined in the context of conditional volatility using a GARCH framework. Similar to the results of Lamoureux and Lastrapes [1990], the findings show a reduction in the significance and magnitude of the conditional variance equation coefficients, and a reduction in the persistence of variance when trading volume is added as an exogenous variable. Hence, there is prima facie evidence that if trading volume proxies for the rate of information arrival, then ARCH effects and much of the persistence in variance can be explained.

Acknowledgments:

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1. **Introduction**

Market folklore suggests that trading volume is positively related to stock return volatility. While there is substantial anecdotal evidence supporting these links, there is little scientific evidence in this area, particularly in Australia. A major limitation has been the lack of substantial theory linking trading volume directly to stock returns. However, more recently, researchers have examined indirect links through models of information arrival and stock returns. Examples include Admati and Pfleiderer [1988], Barclay, Litzenberger and Warner [1990], Barclay and Warner [1993], Brock and Kleidon [1992], Easley and O'Hara [1987], Foster and Viswanathan [1990], Kyle [1985] and Romer [1993]. These papers are generally based on information economics and tend to be focussed toward micro-structure issues.¹

Karpoff [1986, 1987] attempts to provide a theory which directly links returns with trading volume. Karpoff's model ultimately leads to an asymmetric relationship between volume and price change. Empirical tests have generally supported the model (see Karpoff [1987] and Jain and Joh [1988]). Another model which predicts an asymmetric relationship between trading volume and price changes is that originally proposed by Epps [1975] and developed by Jennings, Starks and Fellingham [1981]. The development of the above-mentioned models centre on differences in the costs of taking various market positions. However, the models are also related to information flow.

The mixture of distributions hypothesis has been offered as an explanation linking price change, volume and the rate of information flow (see Epps and Epps [1976] and Harris

¹ Note that a distinction is usually drawn between categories of traders such as information traders, discretionary liquidity traders and non-discretionary liquidity traders. A problem with this literature is that it is not always consistent. For example, the model of Admati and Pfleiderer [1988] suggests that trading volume and the variance of price changes move together, while Foster and Viswanathan
This hypothesis has also been used to explain the presence of autoregressive conditional heteroscedasticity (ARCH) effects (see Lamoureux and Lastrapes [1990]).

This paper tests both the asymmetric model and the mixture of distributions hypothesis in relation to the Australian market. No previous study has tested the relationship between any function of price change and trading volume in the Australian market. The results indicate strong support for the asymmetric model. Furthermore, the results are also consistent with Lamoureux and Lastrapes [1990] and show that ARCH effects are diminished and persistence in variance is reduced when trading volume is incorporated as an explanatory variable in the general ARCH model. These results have implications for inferring return behaviour from trading volume data.

2. Prior Research

There are a number of empirical papers that provide indirect evidence on the relationship between trading volume and stock returns. It is well documented that returns on the New York Stock Exchange (NYSE) tend to follow a U-shaped pattern during the trading day (Harris [1986, 1989], McInish and Wood [1985, 1990a] and Wood, McInish and Ord [1985]). Intraday volatility also follows a U-shaped pattern (Lockwood and Lin [1990]). Similar results have been reported for the Hong Kong Stock Exchange (Ho and Cheung [1991]), the London Stock Exchange (Yadav and Pope [1992]), the Tokyo Stock Exchange (Chang,
Fukuda, Rhee and Takano [1993]) and the Toronto Stock Exchange (McInish and Wood [1990b]). Furthermore, the futures market (Chan, Chan and Karolyi [1991]) and the options market (Peterson [1990]) also exhibit similar U-shaped patterns in both the level and variance of returns.

Jain and Joh [1988], Wei [1992] and Wood, McInish and Ord [1985] show that trading volume (measured as the number of shares traded) follows a U-shaped pattern during the trading day. Hence, considering the similar patterns observed for both volume and variance, a positive correlation between the variance of returns and trading volume may be inferred. Further support is offered by Harris [1987] who finds a positive correlation between changes in volume (measured as the number of transactions) and changes in squared returns for individual NYSE stocks. This relationship was found to be stronger for interday intervals than intraday intervals. Cornell [1981] also finds a positive correlation between changes in volume and changes in absolute price in various futures market contracts.

Bessembinder and Seguin [1992] document evidence which supports a positive relationship between volume and volatility, however this relationship was significantly weakened by the introduction of futures trading in 1982. Subsequent to the introduction of futures trading, Bessembinder and Seguin report a positive relationship between expected futures trading volume and equity market volatility. In a related study into financial and commodity futures,

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2 Although the Tokyo Stock Exchange index returns and volatility appear to have two U-shaped patterns within the day: one "curve" in the morning trading session and another "curve" in the afternoon trading session.
Bessembinder and Seguin [1993] confirm the positive relationship between volume and volatility and document an asymmetric volatility response to unexpected shocks in trading volume. Positive unexpected shocks to trading volume were found to induce an average increase in volatility of 76 percent, while negative unexpected shocks to trading volume induce a smaller response in volatility.

Schwert [1990] argues that volume induces price changes because price changes are an important input into trading strategies. A belief in price persistence will result in many investors wishing to trade in the same direction when there is a price movement. This "herd" mentality becomes a self-fulfilling prophecy as the increased trading exacerbates the change in price which in turn influences more investors to trade in the same direction. However, this argument relies upon price persistence which implies that a random walk in price changes is invalid.

Informed traders will transact when new information (both public and private) becomes available. However, trading based on private information is difficult to identify, and hence trading volume has generally been examined in the context of public information. Woodruff and Senchack [1988] find a high level of volume (measured by both number of stocks traded and number of transactions) immediately following earnings announcements. Similar results have been reported by Brown, Clinch and Foster [1992], Cready and Mynatt [1991], Kiger [1972] and Morse [1981]. This high level of volume disappears quickly (within the first hour).

\[^3\] Although Peterson [1990] reports some evidence of a different pattern in put option returns (but not variance) from the pattern in stock returns and call option returns.
Similarly, Barclay and Litzenberger [1988] find that announcements of new equity issues are associated with large price declines and an abnormally high level of volume (measured by the number of transactions). In contrast, Jain [1988] reports that while S&P 500 Index returns respond rapidly to macro-economic news announcements such as the money supply, consumer price index, industrial production and unemployment statistics, trading volume is unaffected by these announcements. Hence, Jain's results imply that there is no direct association between trading volume and returns.

Of note is that Woodruff and Senchack [1988] find that bad news earnings announcements are associated with a smaller number of transactions but that the average volume per transaction is larger than trades following good news earnings announcements. This result indicates that different measures of volume can provide conflicting results, and goes some way to explaining the inconsistent findings.

French and Roll [1986] show that volatility is higher during trading hours. On an equivalent hourly basis, French and Roll document that volatility during trading hours on the NYSE is far greater than during weekend non-trading hours and conclude that the greater variance during trading time is due to the arrival of private (rather than public) information. Supportive evidence can be found in Oldfield and Rogalski [1980] and Stoll and Whaley [1990]. Houston and Ryngaert [1992] show that market closures during the week affect the pattern of volume and volatility during the week but that the total volume and volatility over the week is constant.
Karpoff [1987] concludes from a review of prior empirical literature that volume and changes in absolute returns are positively associated, but that this association weakens as the measurement interval shortens. Karpoff also concludes that there is only weak evidence supporting a relationship between volume and price change \textit{per se}. Using this evidence as a base, Karpoff [1986, 1987] develops a theoretical model linking returns and trading volume.

Karpoff's [1986] initial model concludes that trading volume is influenced by two mechanisms. To explain the model, denote $i$ as a seller and $j$ as a buyer.\footnote{The following discussion is heavily based on Karpoff [1986].} In equilibrium, the seller's demand price must exceed the buyer's demand price such that $p_i > p_j$. A trade will then occur in the next period ($t=1$) if the change in the buyer's demand price ($\delta_{jt}$) exceeds the change in the seller's demand price ($\delta_{it}$) by an amount sufficient to offset the demand price differential at $t=0$. Thus, a trade will occur in $t=1$ \textit{iff}:

$$p_{jt} \geq p_{it}$$

or $$p_{it} + \delta_{jt} \geq p_{i0} + \delta_{it}$$

or $$\delta_{jt} - \delta_{it} \geq p_{i0} - p_{j0}$$

The net price change for a general investor ($k$) will appear as $\delta_{kt}$ ($\delta_{kt} = \delta_{jt} - \delta_{it}$). If the revision in demand prices follows a stochastic process with mean $\mu$ and variance $\sigma^2$, then:

$$\delta_{kt} = \mu_k + \sigma \varepsilon_k$$

where $\varepsilon_k$ is a zero-mean variable and is independent across investors such that $E(\varepsilon_k \varepsilon_n) = 0$ for all $k \neq h$. 
Thus, the net price revision has two components. First, there is a demand price revision incorporated in the mean $\mu_k$ and secondly, there is an investor specific idiosyncratic term $\varepsilon_k$ which captures changes in individual investor expectations and liquidity desires. In the absence of any new public information, $\mu_k$ is the expected return on the stock. Hence, for any pair of buyers and sellers:

$$\theta = \delta_j - \delta_i = (\mu_j - \mu_i) + \sigma (\varepsilon_j - \varepsilon_i)$$

$$\mu_0 = E(\theta) = \mu_j - \mu_i$$

$$\sigma_\theta^2 = E(\theta - \mu_0)^2 = 2\sigma^2$$

Thus, trades will occur because of movements in $\mu_\theta$, or $\sigma_\theta^2$ or a combination of both. This model leads to a number of predictions.

First, in the absence of any new information, trading will occur because of individual investor idiosyncratic adjustments (i.e. $\sigma_\varepsilon_k > 0$). As long as one investor makes such an adjustment, expected trading volume is positive. Second, trading volume increases proportionately with the number of stock holders such that trading volume is expected to be greater in larger markets. Third, the introduction of transaction costs (including bid-ask spreads) reduces expected trading volume as the change in demand prices ($\delta_j - \delta_i$) must now exceed the original price difference ($p_{i0} - p_{j0}$) plus the transaction costs. Fourth, information arrival may have a mean effect on demand prices but may be interpreted differently by investors such that $\sigma_\theta^2$ increases leading to an increase in trading volume. Fifth, information may have a different effect on the mean revision price between buyers and sellers such that $\mu_j \neq \mu_i$. With constant $\sigma_\theta^2$, trading volume increases if $\mu_j > \mu_i$, but decreases if $\mu_j < \mu_i$. Karpoff's example of this
circumstance involves current owners (or sellers) having strong beliefs about the probability of a takeover offer such that their price revision is relatively small once the offer is announced compared to buyers who had relatively weak beliefs about the probability of a forthcoming offer. Finally, there could be simultaneous changes in \( \mu_\theta \) and \( \sigma_\theta^2 \). Information could have different effects on the mean price response between sellers and buyers but heterogeneous beliefs within each of these groups affects \( \sigma_\theta^2 \). Trading volume will increase if both \( \mu_\theta \) and \( \sigma_\theta^2 \) increase. However, there is no clear effect on trading volume if \( \mu_\theta \) decreases and \( \sigma_\theta^2 \) increases.

The above model assumes that short sales are not possible. However, short selling can be incorporated into the model which results in an asymmetric relationship between volume and price change. If short positions are more costly than long positions, then investors require a greater demand price revision to transact in short positions. Hence, investors in short positions will be less responsive to price changes than investors in long positions. This result leads to an expectation that the association between volume and positive price changes will be greater than the association between volume and negative price changes.

Also note that short selling can only be initiated on a zero-tick in Australia whereby the sale price is at least equal to the last traded price.\(^5\) Hence, there is a lower number of potential traders in the market on down-ticks because of the restriction on short-selling. Therefore, \textit{a priori}, we may expect greater volume on zero- or up-ticks (i.e. on non-negative returns).

\(^5\) See s.846 of the Corporations Law for further details.
Indeed, there is some empirical support for this relationship (see Karpoff [1987] and Jain and Joh [1988]).

Another model which predicts an asymmetric relationship between trading volume and price changes is that originally proposed by Epps [1975] and developed by Jennings, Starks and Fellingham [1981]. In this model, investors are classified as either "optimists" or "pessimists". Again, short positions are assumed to be more costly than long positions. In such a market, investors with short positions would be less responsive to price changes. Jennings, Starks and Fellingham show that (generally) when the trader is a pessimist, the trading volume is less than when the trader is an optimist. Since prices decrease with a pessimistic seller and increase with an optimistic buyer, it follows that volume is low when prices decrease and high when prices increase. As Karpoff [1987] notes, this model relies upon a distinction between optimists and pessimists and the consequent behavioural distinction between the two groups.\(^6\)

The above models are clearly related to information flow. The mixture of distributions hypothesis has been offered as an explanation linking price change, volume and the rate of information flow (see Epps and Epps [1976] and Harris [1987]). Assume that prices and volume react to pieces of information which arrive throughout the trading day. Hence, the daily price change (and volume) is the sum of the intraday price changes (and volume). To the extent that the number of traders with private information changes over time, volatility during

\(^6\) Other behavioural aspects can lead to differences in the volume-price change relationship. For example, if investors are reluctant to realise losses and are more likely to take profits, then volume in a bear market is likely to be lower than volume in a bull market. However, these behavioural features are not easily captured in a formal model.
trading hours is expected to change over time. Further assume that the amount (and rate of arrival) of information varies across days and that price changes and volume are jointly independently and identically distributed with finite variance. Then, this joint distribution will be bivariate normal following the Central Limit Theorem. The daily returns and volume are drawn from a mixture of distributions as the amount (and rate of arrival) of information varies. The mixing variable is the information which arrives. Thus, because of the variation in information arrival, a test of the unconditional distribution of returns will reject normality, while the conditional (upon information) distribution will be normal. The use of volume as a proxy for the mixing variable then provides an indirect test for the link between price changes and information flow. Of course, failure to support such a relationship could be due to failure of the proxy and/or failure of the hypothesis.

In the context of the ARCH class of models, if information arrival is serially correlated and the mixture model holds, then innovations in the information process will lead to momentum in the squared daily returns (see Lamoureux and Lastrapes [1990]). Hence, ARCH effects could result from this process.\(^7\) Thus, if this hypothesis is correct then using volume as the mixing variable and incorporating it in the conditional variance equation of the ARCH process may remove the ARCH effects. This approach is adopted by Lamoureux and Lastrapes [1990] in their examination of 20 US stocks. Their results show that the addition of trading volume (measured as number of shares traded per day) as an exogenous variable to the conditional

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\(^7\) While the trading process is an obvious source of time-varying volatility, there are other hypothesised sources of ARCH as discussed by Bollerslev, Chou and Kroner [1992]. However, the evidence has generally been unsupportive of these other factors (see Baillie, Bollerslev and Redfearn [1993], Engle, Ito and Lin [1990], Engle and Ng [1993] and Laux and Ng [1993]).
variance equation removes the significance of the $\alpha_1$ and $\beta_1$ estimates in the GARCH(1,1) model. Hence, the persistence in volatility is greatly reduced. This finding implies that trading volume is a good alternative for the GARCH process.\textsuperscript{8,9}

Research outside the USA is limited. No study has tested the relationship between any function of price change and trading volume in the Australian market. This paper provides such a test. Furthermore, the paper examines the impact of trading volume on conditional volatility in the context of ARCH following the approach of Lamoureux and Lastrapes [1990].

3. Models

Given the inconsistency in the measurement of trading volume and the inconsistent results of previous research which have employed different measures of trading volume, daily trading volume is measured three ways:

- the daily number of equity trades;
- the daily number of shares traded;
- the daily total dollar value of shares traded.

\textsuperscript{8} The methodology of Lamoureux and Lastrapes [1990] relies upon the assumption that volume is strictly exogenous to the ARCH process. In a more recent paper, Lamoureux and Lastrapes [1994] relax the assumption of trading volume exogeneity through the use of a latent common factor which restricts the joint density of returns and trading volume. They find that the model cannot fully account for the ARCH effects.

\textsuperscript{9} Locke and Sayers [1993] examine a similar issue in the context of the S&P 500 futures market. Using intraday data and a range of variables which proxy for the information arrival variable such as contract volume, floor transactions, the number of price changes and executed order imbalance, Locke and Sayers are unable to remove the persistence in variance. Hence, contrary to Lamoureux and Lastrapes, they conclude that trading volume \textit{per se} cannot explain variance persistence.
The methodology involves testing the relationship between different measures of price change and trading volume. This is initially conducted using standard OLS regressions which test the following relationships:

\[ V_t = \alpha_0 + \gamma_1 |r_t| + \gamma_2 D_t |r_t| + \mu_t \]

...(1) \ Error! Switch argument not specified.

\[ V_t = \alpha_1 + \gamma_3 r_t^2 + \gamma_4 D_t r_t^2 + \mu_t \]

...(2) \ Error! Switch argument not specified.

where: \( V_t \) is the daily measure of volume; \( r_t \) is the daily return; \( D_t = 1 \) if \( r_t < 0 \), and \( D_t = 0 \) if \( r_t \geq 0 \).

The estimate of \( \gamma_1 \) measures the relationship between price change and volume irrespective of the direction of price change. The estimate of \( \gamma_2 \) allows for asymmetry in the relationship. A statistically significant negative value of \( \gamma_2 \) would indicate that the response slope for negative returns is smaller than the response slope for non-negative returns. Such a result would be consistent with an asymmetric relationship.

The second regression model given by expression (2) repeats the analysis for squared returns. Interpretation of the coefficient estimates is as above. In both expressions (1) and (2), the price change variables are a crude measure of volatility. Hence, the regressions test for a relationship between volume and volatility. The results of these regressions are presented in section 5.

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10 Expressions (1) and (2) are modified versions of equations proposed by Jain and Joh [1988].
The second test in this paper examines the effect of trading volume on conditional volatility. This is examined through modification of the GARCH model following the methodology of Lamoureux and Lastrapes [1990]. The basic GARCH model is modified, \( \text{viz}^{11} \)

\[
    r_t = \gamma_0 + \gamma_1 r_{t-1} + \varepsilon_t \text{Switch argument not specified} \ldots (3) \text{Error!}
\]

Switch argument not specified.

where \( \varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t) \)

\[
    h_t = \omega + \beta_1 h_{t-1} + \alpha_1 \varepsilon_{t-1}^2 + \lambda_1 V_t \text{Switch argument not specified} \ldots (4) \text{Error!}
\]

where \( \Omega_{t-1} \) is the information set available at period \( t-1 \).

The GARCH(1,1) model is used for comparison with Lamoureux and Lastrapes [1990]. The significance of the coefficient estimate (\( \lambda_1 \)) indicates the influence of trading volume.

4. Data and Descriptive Statistics

Historical trading volume statistics are difficult to obtain in Australia. However, trading volume on the All Ordinaries Index (AOI) can be accessed through official stock exchange records.\(^{12}\) For the purposes of this paper, daily AOI index values and volume statistics were hand collected from stock exchange records from April 1989, which is the first date at which

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\(^{11}\) The AR(1) process is used in the conditional mean to account for the first-order autocorrelation in the return series.

\(^{12}\) The AOI is recognised as the leading indicator of the Australian stock market. The index comprises the top 250 (approximately) stocks and covers 85-90% of total market capitalisation.
daily volume measures are available. The resultant sample covers the period 24 April 1989 to 31 December 1993.

Table 1 presents summary statistics on the continuously compounded return series, and Table 2 reports the partial autocorrelation estimates.

**Table 1**

**Summary Statistics of the Percentage Daily AOI Return Series:**

<table>
<thead>
<tr>
<th>April 1989 to Dec 1993</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>1,189</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q(20)</th>
<th>Q^2(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.42</td>
<td>46.90</td>
</tr>
</tbody>
</table>

| a | t-statistic for difference from zero. |
| b | z-statistic for difference from zero. |
| c | z-statistic for difference from zero. |
| d | The Q(20) statistic is the Box-Ljung portmanteau test for first to twentieth-order autocorrelation in the return series and is distributed as χ^2(20). Probability value in parentheses. |
| e | The Q^2(20) statistic is the Box-Ljung portmanteau test for first to twentieth-order autocorrelation in the squared return series and is distributed as χ^2(20). Probability value in parentheses. |

**Table 2**

**Partial Autocorrelation Estimates for the Daily Rate of Return**

<table>
<thead>
<tr>
<th>AOI: April 1989 to December 1993</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag (days)</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Estimate</td>
</tr>
<tr>
<td>Std errors</td>
</tr>
</tbody>
</table>

| a | Number of standard errors from zero. |

---

Index values on the All Ordinaries Accumulation Index were collected. The analysis was repeated using the All Ordinaries Price Index and similar results were obtained.
From Table 2, there is significant first-order autocorrelation which is likely to be substantially induced by thin trading in many of the index stocks. However, there is generally little evidence of autocorrelation at lags higher than one. In the GARCH model, an AR(1) process is used in the conditional mean to account for the first-order autocorrelation. The results from the LM test (Engle [1982]) for ARCH in the return series after purging by an AR(1) process indicate significant ARCH errors at all lags.

Summary statistics on trading volume are presented in Table 3. Three measures of trading volume are examined which are the daily number of transactions, the daily total number of shares traded and the daily total dollar value of shares traded.

Table 3


<table>
<thead>
<tr>
<th></th>
<th>Trades&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Volume&lt;sup&gt;b&lt;/sup&gt; ('000)</th>
<th>Value&lt;sup&gt;c&lt;/sup&gt; ($'000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>8,215</td>
<td>131,246</td>
<td>264,459</td>
</tr>
<tr>
<td>Median</td>
<td>7,276</td>
<td>110,287</td>
<td>231,292</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3,195.6</td>
<td>1,992.4</td>
<td>3,831.9</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.5293</td>
<td>1.8609</td>
<td>2.1469</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.2723</td>
<td>4.7782</td>
<td>7.6599</td>
</tr>
<tr>
<td>Minimum</td>
<td>3,018</td>
<td>39,754</td>
<td>52,547</td>
</tr>
<tr>
<td>Maximum</td>
<td>21,953</td>
<td>550,295</td>
<td>1,288,045</td>
</tr>
</tbody>
</table>

Notes:  
<sup>a</sup> Daily number of share transactions.

<sup>14</sup> The All Ordinaries Index comprises about 250 stocks, some of which do not trade regularly on a daily basis.

<sup>15</sup> For example, the calculated test statistics for lags 1, 2, 5 and 10 are 69.75, 69.99, 69.55 and 72.47, respectively.
Table 4 presents the Pearson correlation matrix between the three measures of daily trading volume. The three measures of volume are closely related. The highest correlation is between the number of shares traded and the dollar value, while the lowest (but still highly significant) correlation is between the number of transactions and dollar value.

Table 4

<table>
<thead>
<tr>
<th></th>
<th>Trades</th>
<th>Volume</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trades(^a)</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume(^b)</td>
<td>0.8022(^*)</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>Value(^c)</td>
<td>0.6676(^*)</td>
<td>0.8230(^*)</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Notes:  
\(^*\) Significant at the 0.001 level.  
\(^a\) Daily number of share transactions.  
\(^b\) Daily number of shares traded.  
\(^c\) Daily dollar value of shares traded.

Table 5 presents the mean daily measures of trading volume across days of the week. Results of F-tests which test for the equality of the mean trading volume measures across days of the week are presented in the last row of the table. These results indicate that the null hypothesis of equality of mean daily trading volume across days of the week can be rejected at standard significance levels. The mean trading volume is consistently lowest on Mondays and generally rises through the week. This feature is present in all three measures of trading volume.
Table 5
Mean Daily Trading Volume Measures Across Days of the Week

<table>
<thead>
<tr>
<th></th>
<th>Trades(^a)</th>
<th>Volume(^b) (‘000)</th>
<th>Value(^c) ($’000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>7,689</td>
<td>111,296</td>
<td>214,669</td>
</tr>
<tr>
<td>Tuesday</td>
<td>8,272</td>
<td>124,352</td>
<td>243,860</td>
</tr>
<tr>
<td>Wednesday</td>
<td>8,314</td>
<td>133,219</td>
<td>268,035</td>
</tr>
<tr>
<td>Thursday</td>
<td>8,405</td>
<td>142,423</td>
<td>286,578</td>
</tr>
<tr>
<td>Friday</td>
<td>8,393</td>
<td>144,468</td>
<td>309,864</td>
</tr>
<tr>
<td>F-test(^d) (prob. value)</td>
<td>1.993 (0.09)</td>
<td>9.469 (0.00)</td>
<td>18.666 (0.00)</td>
</tr>
</tbody>
</table>

Notes:
\(^a\) Daily number of share transactions.
\(^b\) Daily number of shares traded.
\(^c\) Daily dollar value of shares traded.
\(^d\) F-test for the equality of mean daily trading volume across days of the week.

5. Trading Volume and Price Movements

This section presents the empirical results of the test of the relationship between trading volume and price movements (i.e. expressions (1) and (2)).\(^16\) The measures of trading volume are standardised.\(^17\),\(^18\) Expression (1) tests for a relationship between trading volume and absolute returns. The OLS regression results are reported in Table 6.

---

\(^{16}\) All regressions in this section were repeated using first differences of the trading volume measures as the independent variable. The sign and significance of the coefficient estimates from these regressions are all very similar to those reported in the text except for the case of \(V_t = \text{Volume}\). In this regression, the absolute value of \(\gamma_1 (\gamma_3)\) exceeded \(\gamma_2 (\gamma_4)\) which removes the "anomaly" from Table 6.

\(^{17}\) Daily trading volume was standardised by subtracting the mean and dividing by the standard deviation of the respective trading volume measure.

\(^{18}\) The OLS regressions were also run without standardising trading volume. By definition, the significance of the parameter estimates (except the constant) must be identical to those reported in the text, and this was confirmed empirically.
Table 6
Relationship Between Standardised Trading Volume and Absolute Returns

<table>
<thead>
<tr>
<th></th>
<th>( V_t = \text{Trades}^b )</th>
<th>( V_t = \text{Volume}^c )</th>
<th>( V_t = \text{Value}^d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 ) (t-statistic)(^e)</td>
<td>-0.0943 (-2.18)(^*)</td>
<td>-0.0193 (-0.46)</td>
<td>-0.0962 (-2.42)(^*)</td>
</tr>
<tr>
<td>( \gamma_1 ) (t-statistic)(^e)</td>
<td>24.9061 (3.53)(^*)</td>
<td>13.2838 (1.96)(^*)</td>
<td>27.8956 (4.13)(^*)</td>
</tr>
<tr>
<td>( \gamma_2 ) (t-statistic)(^e)</td>
<td>-20.7098 (-2.75)(^*)</td>
<td>-22.2224 (-3.12)(^*)</td>
<td>-26.6677 (-3.81)(^*)</td>
</tr>
<tr>
<td>F-test (prob. value)</td>
<td>8.073 (0.000)(^*)</td>
<td>5.054 (0.007)(^*)</td>
<td>10.924 (0.000)(^*)</td>
</tr>
</tbody>
</table>

Notes:
\(^*\) Significant at the 0.05 level using a two-tailed test.
\(^a\) The results are from the following OLS regression:
\[ V_t = \alpha_0 + \gamma_1 |r_t| + \gamma_2 D_t |r_t| + \mu, \]
where: \( V_t \) is the standardised daily measure of volume; \( r_t \) is the daily return;
\( D_t = 1 \) if \( r_t < 0 \), and \( D_t = 0 \) if \( r_t \geq 0 \).
\(^b\) Daily number of share transactions.
\(^c\) Daily number of shares traded.
\(^d\) Daily dollar value of shares traded.
\(^e\) Standard errors are computed using White's [1980] heteroscedastic consistent variance-covariance matrix.

The results in Table 6 indicate strong support for the model. The estimates of \( \gamma_1 \), which measure the relationship between price change and volume irrespective of the direction of the price change, are significantly positive across all three measures of trading volume. Similarly, the estimates of \( \gamma_2 \), which allow for asymmetry in the relationship, are also significant across all three measures of trading volume. The negative value of \( \gamma_2 \) indicates that the slope for negative returns is smaller than the slope for positive returns. That is, \( \gamma_2 \) is a measure of the difference in the slope coefficient of the trading volume and returns relationship between non-negative and negative returns. For instance, the slope coefficient of the relationship for negative returns is 4.1981, -8.9386 and 1.2279 for the three measures of trading volume,
respectively. Interestingly, the second measure (number of shares traded) has a negative slope coefficient implying that for negative price movements, the standardised level of trading volume declines with the absolute magnitude of the price change.\footnote{Refer to footnote 16.}

Expression (2) uses squared returns instead of absolute returns as the price change measure. Thus, the expression tests for a relationship between daily trading volume and an alternative specification of "raw" volatility. The results from this OLS regression are reported in Table 7. The significance of the results is weaker when squared returns are used and for one measure of trading volume (number of shares traded), there is no evidence of a significant relationship.

The use of dollar value of traded shares as the trading volume measure yields results which are consistent with an asymmetric relationship. The difference in the results between Tables 6 and 7 is due to differences in the measure of "raw" volatility.
Table 7
Relationship Between Standardised Trading Volume and Squared Returns\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th>$V_t = \text{Trades}^b$</th>
<th>$V_t = \text{Volume}^c$</th>
<th>$V_t = \text{Value}^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.0356</td>
<td>-0.0107</td>
<td>-0.0378</td>
</tr>
<tr>
<td>(t-statistic)\textsuperscript{e}</td>
<td>(-1.17)</td>
<td>(-0.35)</td>
<td>(-1.25)</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>810.7654 (2.32)*</td>
<td>347.8148 (1.17)</td>
<td>998.2623 (3.18)*</td>
</tr>
<tr>
<td>(t-statistic)\textsuperscript{e}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>-600.3210 (-1.71)</td>
<td>-396.4431 (-1.28)</td>
<td>-926.7568 (-2.92)*</td>
</tr>
<tr>
<td>(t-statistic)\textsuperscript{e}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-test</td>
<td>4.323 (0.014)*</td>
<td>0.695 (0.499)</td>
<td>5.062 (0.007)*</td>
</tr>
<tr>
<td>(prob. value)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:  
\textsuperscript{a} Significant at the 0.05 level using a two-tailed test.  
\textsuperscript{b} The results are from the following OLS regression:  
\[ V_t = \alpha_1 + \gamma_3 r_t^2 + \gamma_4 D_t r_t^2 + \mu_t \]

\textsuperscript{c} where: $V_t$ is the standardised daily measure of volume; $r_t$ is the daily return;  
\[ D_t = 1 \text{ if } r_t < 0, \text{ and } D_t = 0 \text{ if } r_t \geq 0. \]

\textsuperscript{d} Daily number of share transactions.  
\textsuperscript{e} Daily number of shares traded.  
\textsuperscript{f} Daily dollar value of shares traded.  
\textsuperscript{g} Standard errors are computed using White's [1980] heteroscedastic consistent variance-covariance matrix.

The effect of past measures of price change and trading volume on the relationship was also examined by the addition of lagged values of the explanatory variables and lagged trading volume as an explanatory variable to expressions (1) and (2). Only lagged trading volume proved to be significant. While this variable was highly significant in all model specifications, its addition had only a marginal impact on the significance on the existing explanatory variables.

In summary, these results give support to the asymmetric trading volume-returns relationship. In bull markets, relatively heavier trading volume is associated with price changes of the same
absolute magnitude as compared to bear markets. Karpoff [1987] attributes this asymmetry to the greater cost of short positions. In Australia, tight restrictions on short selling (implying greater costs) on the ASX may strengthen Karpoff’s argument. However, while Karpoff (and others) assume that short selling is possible, but costly, the ASX regulations limit short selling to approved stocks under certain conditions and thus, the hypothesis can apply only to these stocks. Nevertheless, these stocks generally comprise the All Ordinaries Index.

6. Trading Volume on Individual Stocks

An extension to the previous section is to examine trading volume at the individual stock level. Historical series of both returns and volume are difficult to obtain in Australia, particularly on a daily basis. Hence, this analysis is limited to five stocks. The stocks were selected on the basis of the top five ranked Australian stocks by market capitalisation. Daily prices, volume, dividend and capitalisation information were collected and a daily returns and standardised volume series were constructed. For each stock, the sample period commenced in November 1986 and contains 1,958 daily observations.

Tables 8 and 9 present the coefficient estimates of equations (1) and (2) and correspond to the aggregate market results reported in Tables 6 and 7.

---

20 Volume is the number of shares traded in each day and is standardised by subtracting the mean for that company and dividing by the standard deviation of the sample.
Table 8
Relationship Between Standardised Trading Volume and Absolute Returns on Individual Stocks\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>BHP</th>
<th>BTR</th>
<th>CRA</th>
<th>NAB</th>
<th>NCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_0) (t-statistic)(^b)</td>
<td>-0.0001 (3.64)*</td>
<td>-0.0001 (1.95)</td>
<td>-0.0001 (2.63)*</td>
<td>-0.0001 (3.11)*</td>
<td>-0.0001 (5.46)*</td>
</tr>
<tr>
<td>(\gamma_1) (t-statistic)(^b)</td>
<td>0.0082 (3.43)*</td>
<td>0.0071 (3.54)*</td>
<td>0.0075 (5.18)*</td>
<td>0.0071 (3.42)*</td>
<td>0.0113 (7.14)*</td>
</tr>
<tr>
<td>(\gamma_2) (t-statistic)(^b)</td>
<td>0.0061 (1.70)</td>
<td>-0.0058 (-2.90)*</td>
<td>-0.0027 (-1.41)</td>
<td>0.0034 (1.01)</td>
<td>-0.0046 (-2.45)*</td>
</tr>
<tr>
<td>F-test (prob. value)</td>
<td>15.721 (0.001)*</td>
<td>7.182 (0.001)*</td>
<td>9.894 (0.001)*</td>
<td>11.392 (0.001)*</td>
<td>44.853 (0.001)*</td>
</tr>
</tbody>
</table>

Notes:
- * Significant at the 0.05 level using a two-tailed test.
- \(^a\) The results are from the following OLS regression:
  \[ V_t = \alpha_0 + \gamma_1 |r_t| + \gamma_2 D_t |r_t| + \mu_t \]
  where: \(V_t\) is the standardised daily measure of volume; \(r_t\) is the daily return; and
  \(D_t = 1\) if \(r_t < 0\), and \(D_t = 0\) if \(r_t \geq 0\).
Table 9
Relationship Between Standardised Trading Volume and Squared Returns on Individual Stocks\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>BHP</th>
<th>BTR</th>
<th>CRA</th>
<th>NAB</th>
<th>NCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_t) \hspace{1cm} (t-statistic) (^b)</td>
<td>-0.0001 (-0.97)</td>
<td>-0.0001 (-0.73)</td>
<td>-0.0001 (-0.66)</td>
<td>-0.0001 (-0.50)</td>
<td>-0.0001 (-0.89)</td>
</tr>
<tr>
<td>(\gamma_3) \hspace{1cm} (t-statistic) (^b)</td>
<td>0.1224 (2.61)*</td>
<td>0.0804 (2.63)*</td>
<td>0.0709 (4.17)*</td>
<td>0.0587 (1.66)</td>
<td>0.0385 (2.58)*</td>
</tr>
<tr>
<td>(\gamma_4) \hspace{1cm} (t-statistic) (^b)</td>
<td>-0.0119 (-0.23)</td>
<td>-0.0804 (-2.64)*</td>
<td>-0.0654 (-3.78)*</td>
<td>-0.0134 (-0.31)</td>
<td>-0.0252 (-1.56)</td>
</tr>
<tr>
<td>F-test \hspace{1cm} (prob. value)</td>
<td>12.035 (0.001)*</td>
<td>4.506 (0.011)*</td>
<td>4.400 (0.012)*</td>
<td>3.114 (0.045)*</td>
<td>15.355 (0.001)*</td>
</tr>
</tbody>
</table>

Notes:  
\(^a\) The results are from the following OLS regression:
\[ V_t = \alpha_t + \gamma_3 r_t^2 + \gamma_4 D_t r_t^2 + \mu_t \]
where: \(V_t\) is the standardised daily measure of volume; \(r_t\) is the daily return; and \(D_t = 1\) if \(r_t < 0\), and \(D_t = 0\) if \(r_t \geq 0\).

\(^b\) Standard errors are computed using White's [1980] heteroscedastic consistent variance-covariance matrix.

The results in Table 8 are generally supportive of the aggregate market results. That is, the estimates of \(\gamma_1\), which measure the relationship between price change and volume irrespective of the direction of the price change, are significant across all individual stocks. The estimates of \(\gamma_2\), which allow for asymmetry in the relationship, are significant (and negative) for BTR (BTR-Nylex) and NCP (News Corporation). The slope for negative returns is smaller than the slope for positive returns and is positive for every stock.

There is some variation across the stocks in terms of the magnitude and significance of the coefficient estimates. The strongest results are obtained for NCP which is well-known in
market circles as a highly volatile stock, while the weakest results are obtained for BHP (Broken-Hill Proprietary Co.) and NAB (National Australia Bank) which are known as low-volatility stocks.

The results in Table 9 are also generally supportive of the aggregate market results. The estimates of $\gamma_3$ are significant for all stocks except NAB, while the estimates of $\gamma_4$ are significant for BTR and CRA. Consistent with the earlier analysis, the use of squared returns instead of absolute returns appears to weaken the general results.

7. Trading Volume and Conditional Volatility

This section examines the relationship between trading volume and conditional volatility of the aggregate market by modifying the conditional variance equation of the GARCH model to include trading volume as an explanatory variable. This approach closely follows Lamoureux and Lastrapes [1990]. The exact specification of the GARCH model is given by expressions (3) and (4) which are the conditional mean and conditional variance equations, respectively.\(^{21}\)

Again, three measures of standardised trading volume ($V_t$) for the aggregate market are used. First, a GARCH(1,1) model is estimated without trading volume which results in the following:\(^{22}\)

---

\(^{21}\) A GARCH(1,1) model is used following Lamoureux and Lastrapes [1990] and for reasons of parsimony.

\(^{22}\) GARCH model estimates were obtained using the Berndt, Hall, Hall and Hausman [1974] algorithm employing numerical derivatives.
\[
    r_t = 0.0002 + 0.1805 r_{t-1} + \varepsilon_t
    \]
    \begin{align*}
    (0.92) & \quad (5.16) \\
    \text{where } \varepsilon_t & \mid \Omega_{t-1} \sim N(0, h_t) \\
    \end{align*}
\]

\[
    h_t = 0.0001 + 0.1957 \varepsilon_{t-1}^2 + 0.3218 h_{t-1}
    \]
    \begin{align*}
    (4.04) & \quad (4.56) & \quad (2.19) \\
    \text{where } \varepsilon_t & \mid \Omega_{t-1} \sim N(0, h_t) \\
    \end{align*}
\]

Diagnostic tests of the standardised residuals using the sign and size bias tests of Engle and Ng [1993] indicate a good fit and no sign or size biases. Thus, an asymmetric GARCH model is not required.

Next, a modified GARCH(1,1) model is estimated using the standardised trading volume as an explanatory variable. The results from using the standardised daily number of trades is:\[23\]

\[
    r_t = 0.0002 + 0.1689 r_{t-1} + \varepsilon_t
    \]
    \begin{align*}
    (0.76) & \quad (5.07) \\
    \text{where } \varepsilon_t & \mid \Omega_{t-1} \sim N(0, h_t) \\
    \end{align*}
\]

\[
    h_t = 0.0001 + 0.1915 \varepsilon_{t-1}^2 + 0.1234 h_{t-1} + 0.1116 V_t
    \]
    \begin{align*}
    (6.97) & \quad (4.10) & \quad (1.15) & \quad (6.90) \\
    \text{where } \varepsilon_t & \mid \Omega_{t-1} \sim N(0, h_t) \\
    \end{align*}
\]

\[23\] The other two measures of trading volume were also separately added to modify the GARCH conditional variance equation and qualitatively similar results were obtained.
Diagnostic tests of the standardised residuals from this estimated model again indicate a good fit and no sign or size biases. The main feature of the estimated model is the significance of the coefficient on trading volume and the insignificance of the coefficient on lagged conditional volatility. Similar to the results of Lamoureux and Lastrapes [1990], there is a reduction in the significance and magnitude of the GARCH coefficients. The persistence in variance (as measured by the sum of $\alpha_1$ and $\beta_1$) falls by almost 40 percent from 0.52 to 0.32.

A likelihood ratio test which compares the restricted (standard GARCH(1,1)) model with the unrestricted (modified GARCH(1,1) with trading volume) model yields a test statistic of 13.61 ($\chi^2(1)$) which is significant at the 0.001 level, thereby favouring the unrestricted (trading volume) model. Thus, there is prima facie evidence that if trading volume proxies for the rate of information arrival, then ARCH effects and much of the persistence in variance can be explained.

However, as noted by Lamoureux and Lastrapes [1990] and discussed previously, if trading volume is not strictly exogenous, then there is possible simultaneity bias. An alternative specification of expression (4) was run to remove this potential bias which involved using lagged trading volume (i.e. $V_{t-1}$ instead of $V_t$). Lagged trading volume was found to be insignificant. Thus, the reported results must be interpreted cautiously, (also see Lamoureux and Lastrapes [1994]).
8. Summary

This paper has examined the relationship between trading volume and stock market volatility. As this issue had not previously been studied in Australia, the initial analysis centred upon the volume-price change relationship. Evidence was found which supports an asymmetric model. The relationship between price change and volume, irrespective of the direction of the price change, was significant across three measures of daily trading volume for the aggregate market and was significant for individual stocks. Furthermore, evidence was found supporting the hypothesis that the volume-price change slope for negative returns is less steep than the slope for positive returns, thereby supporting the asymmetric relationship.

Trading volume was then examined in the context of conditional volatility using a GARCH framework. Similar to the results of Lamoureux and Lastrapes [1990], the findings show a reduction in the significance and magnitude of the GARCH coefficients, and a reduction in the persistence of variance when trading volume is added as an exogenous variable to the conditional variance. Hence, there is evidence that if trading volume proxies for the rate of information arrival, then ARCH effects and much of the persistence in variance can be explained.

As a caveat, this study is related to information flow. The methodology involved a "macro" approach using fairly crude proxy variables. Thus, the research and results are preliminary. Further work is required in this area, which utilises better proxy variables and is conducted at the intraday level before firm conclusions can be reached.
References


